14th Conference of the European
Society for Fuzzy Logic and Technology

Book of Abstracts

EUSFLAT 2025

Riga, Latvia,
July 21–25, 2025

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14th Conference of the European Society for Fuzzy Logic

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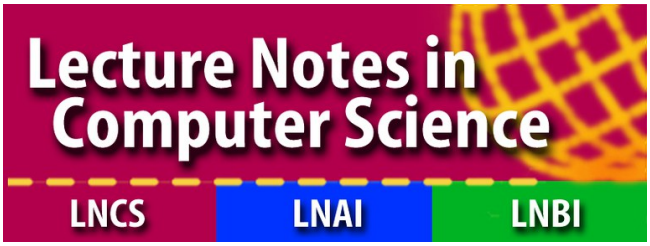

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Preface

We are delighted to present the abstract book of the 14th Conference of the European Society for Fuzzy Logic and Technology (EUSFLAT 2025), which took place in Riga, Latvia, from July 21 to 25, 2025. This biennial conference continued the tradition of bringing together researchers, practitioners, and students from around the world who work in the broad area of fuzzy logic and related fields, including soft computing, computational intelligence, uncertainty modeling, and approximate reasoning. The event served as a platform for the exchange of ideas, dissemination of new results, and the strengthening of collaborations across academic, scientific, and industrial domains.

Nearly 26 years ago, the inaugural EUSFLAT-ESTYLF Joint Conference was convened in Palma de Mallorca from September 22 to 25, 1999. That event marked the first official conference of the European Society for Fuzzy Logic and Technology (EUSFLAT), following its establishment earlier that same year. Since then, the Society has organized successful conferences in Leicester (2001), Zittau (2003), Barcelona (2005), Ostrava (2007), Lisbon (2009), Aix-les-Bains (2011), Milan (2013), Gijón (2015), Warsaw (2017), Prague (2019), Bratislava (2021), and again Palma de Mallorca (2023). The fourteenth edition was hosted in Riga, organized by the University of Latvia, in cooperation with EUSFLAT.

All abstracts in this book were carefully reviewed by the program chairs. As a result of this evaluation process, 72 abstracts were accepted. In addition, the book includes abstracts from 53 accepted papers. Topics covered include, but are not limited to, fuzzy sets and systems, knowledge representation, fuzzy control, decision-making, machine learning, data analysis, and applications in engineering, economics, and the social sciences.

The EUSFLAT conference series has served as a central forum for the community since its inception, promoting interdisciplinary collaboration and addressing new challenges in science and technology through fuzzy logic and soft computing. The 2025 edition stood out not only for the quality of its scientific contributions but also for its location – Riga, a city known for its rich cultural heritage, architectural beauty, and its growing role as a hub for science and innovation in the Baltic region. The Local Organizing Committee worked tirelessly to ensure a warm, efficient, and memorable experience for all participants.

In addition to regular sessions, EUSFLAT 2025 featured invited talks by distinguished researchers, special sessions on emerging topics, tutorials, and panel discussions. These components were designed to offer attendees a well-rounded and intellectually stimulating experience that combined depth with breadth. The program included keynote lectures by:

- Óscar Cordón (University of Granada, Spain),
- Irina Perfilieva (University of Ostrava, Czech Republic),
- Humberto Bustince (Public University of Navarre, Spain),
- Katarzyna Kaczmarek-Majer (Systems Research Institute, Polish Academy of Sciences, Warsaw, Poland),
- Bernard De Baets (Ghent University, Belgium),
- Ulrich Bodenhofer (University of Applied Sciences Upper Austria, Hagenberg, Austria),
- Andris Ambainis (University of Latvia, Latvia).

We are deeply grateful to all authors for submitting their research and sharing their insights with the community. We sincerely thank the General and Program Chairs for their time, expertise, and thoughtful feedback, and we acknowledge the special session organizers for their initiative and commitment. Our appreciation also goes to the invited speakers for their inspiring presentations and to the session chairs for their essential role in ensuring the smooth flow of the technical program.

Last but not least, we would like to dedicate this abstract book to the late Alexander Šostak, who was an exemplary member of our society and a great researcher, and who was very happy and delighted that this fourteenth edition was going to be held in his city.

July 2025

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Part III: EUSFLAT2025 Main Track

Part I
Plenary speakers

Artificial Intelligence for Skeleton-based Biological Profiling and Forensic Human Identification

Oscar Cordón

Dept. of Computer Science and Artificial Intelligence
Andalusian Research Institute on Data Science and Computational Intelligence
University of Granada

Skeleton-based forensic identification methods carried out by anthropologists, odontologists, and pathologists represent the first step in every human identification (ID) process and the victim's last chance for identification when DNA or fingerprints cannot be applied. They include methods as biological profiling (BP), comparative radiography (CR), craniofacial superimposition (CFS), and comparison of dental records. BP involves the study of skeletal remains to find characteristic traits (age, sex, stature, and ancestry) that support determining the identity of the individual. It plays a crucial role in narrowing the range of potential matches during the process of ID, prior to the corroboration by any ID technique. Meanwhile, CFS aims to overlay a skull with some ante-mortem (AM) images of a candidate in order to determine if they correspond to the same person.

However, practitioners still follow an observational paradigm using subjective methods introduced many decades ago; namely, oral description and written documentation of the findings obtained and the manual and visual comparison of AM and post-mortem (PM) data. Designing systematic, automatic, and trustworthy methods to support the forensic anthropologist when applying BP, CFS, CR, and odontogram comparison, avoiding the use of subjective, error-prone and time-consuming manual procedures, is mandatory to enhance forensic ID. The use of artificial intelligence technologies (computational intelligence methods (evolutionary algorithms, fuzzy sets, and deep learning); computer vision (image segmentation and processing, as well as 3D-2D image registration); and explainable machine learning) is a natural way to achieve this aim. This talk is devoted to present some hybrid artificial intelligence systems for CFS, CR, and skeleton-based age-at-death and sex assessment developed in collaboration with the University of Granada's Physical Anthropology Lab within a twenty yearlong research project. Some of those systems are protected by international patents, exploited by Panacea Cooperative Research in the Skeleton-ID software, which is under commercialization in different countries.

Oscar Cordón is a Professor at the University of Granada (UGR), where he was the Founder and former Leader of the Virtual Learning Center (2001–05) and Vice President of Digital University (2015–19). He was one of the Founding Researchers at the European Centre for Soft Computing (2006–11), becoming a Distinguished Affiliated Researcher until 2015. For over 30 years, he has been an internationally recognized contributor to research in artificial intelligence (AI), being included in the Top 2% of the most cited researchers in the world in AI (University of Stanford-Elsevier). He has received different international awards.

His current research focuses on two main areas: AI for forensic identification (in collaboration with the UGR Physical Anthropology Lab and various international forensic laboratories and security agencies), and agent-based modeling and social network analysis for marketing.

Alexander Šostak's Groundbreaking Contributions to Fuzzy Mathematics

Irina Perfilieva

Institute for Research and Applications of Fuzzy Modeling (IRAFM)
Centre of Excellence IT4Innovations, University of Ostrava
Ostrava, Czech Republic

This talk explores Alexander Šostak's foundational contributions to generalized topologies, which extend classical topological structures into more flexible and expressive frameworks. As one of the pioneers in this field, Šostak developed key generalizations—including fuzzy topologies, L-topologies, and intuitionistic fuzzy topologies—enabling richer mathematical modeling of uncertainty, gradation, and imprecision.

His work demonstrates how generalized topologies serve as powerful tools for analyzing complex systems, bridging abstract mathematics and real-world applications in computer science, artificial intelligence, and engineering. Key aspects of his research include:

- Axiomatic foundations of generalized topologies and their compatibility with classical and fuzzy set theory.
- Convergence structures in generalized settings, with applications in functional analysis and beyond.
- Uniform and proximity structures adapted to fuzzy and lattice-based topological spaces.
- Practical applications in measure theory, optimization, and decision-making under uncertainty.

Šostak's legacy lies in transforming generalized topologies into a vital framework for both theoretical advances and interdisciplinary problem-solving.

Irina Perfilieva is a Professor of Applied Mathematics at the University of Ostrava, Czech Republic, where she also leads the Theoretical Research Department at the Institute for Research and Applications of Fuzzy Modeling. She earned her M.S. and Ph.D. degrees in Applied Mathematics from Lomonosov Moscow State University and is internationally recognized for her contributions to fuzzy logic, fuzzy approximation, and fuzzy relation equations. She has published extensively, including over 270 scientific papers and several books, and has held editorial roles in leading journals such as Fuzzy Sets and Systems.

She has served on the board of the European Society for Fuzzy Logic and Technology (EUSFLAT), including two terms as Secretary, and was named an Honorary Member in 2013. She is also a Fellow of the International Fuzzy Systems Association (IFSA). Her research has led to practical applications in time series processing, internet services, and image analysis.

Some Applications of Data Fusion Functions in Artificial Intelligence

Humberto Bustince

Department of Statistics, Informatics and Mathematics
Public University of Navarre (UPNA)
Pamplona, Spain

In most artificial intelligence problems, the fusion of information is a crucial step in order to get appropriate results. In this talk, we focus on recently developed data fusion techniques based or inspired on the well-known Choquet and Sugeno integrals, with an eye kept on possible applications. In particular, we consider how some relevant problems may be addressed in order to get successful applications of these ideas:

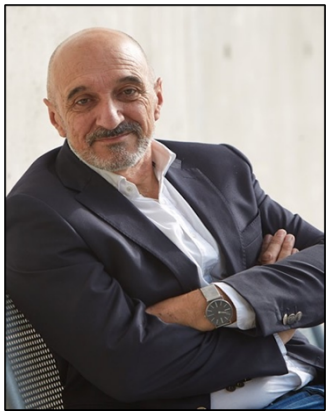
- How can we deal with data representations structures which do not allow for the use of operations such as the substraction? This will lead us to the notion of d-integral.
- And, what about the complexity problem related to the construction of the fuzzy measure?

This will lead us the notion of Choquet-inspired aggregation function. The main core of our talk will be devoted to the use of these theoretical developments in three types of problems, in order to show how fuzzy theory can be an appropriate tool to deal with problems in artificial intelligence:

1. - Deep learning problem. We will show how these functions can be successfully used for modifying the pooling step in Convolutional Neural networks, as well as in LSTM models to get improved results.
2. - The computational brain problem, where we will show how the use of these new functions and a representation of uncertainty of data in terms of extensions of fuzzy sets leads to a significant improvement in the experimental results.
3. - Classification problems, analyzing whether they can be used to improve the results of classical fuzzy rule-based algorithm.

Humberto Bustince is a Full Professor of Computer Science and Artificial Intelligence at the Public University of Navarra and Honorary Professor at the University of Nottingham. He has published over 300 works, introducing key concepts in data fusion and uncertainty modeling in AI, such as ignorance functions, overlap functions, CF-integrals, and d-integrals. He collaborates with leading international research groups and serves as Editor-in-Chief of Mathware & Soft Computing and Axioms.

He is a Fellow of both IEEE and IFSA, serving as IFSA President since 2023. His work has earned him the National Computer Science Prize "José García Santesmases" and the EUSFLAT Scientific Excellence Award (both in 2019), as well as the Cross of Carlos III the Noble from the Government of Navarra. He is also a member of the Basque Academy of Sciences, Arts and Literature, Jakiunde



The Role of Fuzzy Linguistic Summaries - New Methods and Challenges

Katarzyna Kaczmarek-Majer

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Fuzzy linguistic summaries can serve as human-consistent information granules for describing in natural language large datasets, including inhomogeneous time series. We will briefly recall related work on the evaluation criteria for the quality of individual sentences, such as the degrees of truth, confidence, support, informativeness, or focus. However, quality metrics for individual sentences have not always proven sufficient without exposing a clarification for the group, or a sequence, of sentences to explain the broader context. The assessment of the quality of the groups or sequences of summaries becomes even more complex for real-world scenarios in which additional data becomes available over time or the existing information is incomplete. During the presentation, we will concentrate on the consistency and stability of sets of summaries. We will also address challenges related to the construction of linguistic expressions in evolving environments, as well as considerations of human perception of information granules during expert validation. Finally, we will explore how recent developments in the theory of evaluative linguistic expressions and fuzzy association rule mining can complement linguistic summarization. The talk will conclude with a practical example from the remote health monitoring system, which aims to support doctors and patients in monitoring affective episodes and fuzzy linguistic summaries will be its core component.

Katarzyna Kaczmarek-Majer is an Associate Professor at the Systems Research Institute of the Polish Academy of Sciences and the Principal Investigator of the "Explainable Artificial Intelligence for Monitoring Acoustic Features Extracted from Speech" (ExplainMe) project. She is also a contributor to the "Research of Excellence on Digital Technologies and Well-being" (DigiWell) at the Institute for Research and Applications of Fuzzy Modeling, University of Ostrava, Czech Republic. She holds dual M.Sc. degrees in Mathematics and Computer Science from the University of Poznań, and a Ph.D. with distinction in Computer Science from the Systems Research Institute of the Polish Academy of Sciences in Warsaw.

Her research focuses on fuzzy linguistic summarization, computational intelligence, granular computing, and data stream analysis, with applications primarily in medicine and healthcare.

It's All Convolution, Stupid!

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The theory of fuzzy sets, nearing its sixtieth anniversary, faces a challenging landscape marked by a proliferation of generalizations lacking semantic clarity and robust elicitation procedures. Similarly, numerous variants of fuzzy decision-making methods populate the literature, often characterized by ad hoc choices and unproven claims of enhanced decision-making capabilities. These developments not only fail to advance the field but also risk tarnishing its reputation in domains such as machine learning and operations research. Amidst these challenges, it is crucial to revisit foundational contributions that have stood the test of time. The compositional rule of inference and the extension principle introduced by Lotfi Zadeh in the seventies offer enduring insights, albeit often overlooked in contemporary discourse. Additionally, Goguen's early recognition of lattice theory as the appropriate framework for fuzzy set theory remains as relevant today as ever. Drawing from these foundational principles, this talk explores pivotal milestones in fuzzy set theory, including fuzzy relational equations, computation with fuzzy quantities, and convolution lattices. These concepts not only address the propagation of non-stochastic uncertainty but also provide a lens through which to critically assess recent developments in the field. By reexamining these fundamental concepts and their applications, we can navigate the complexities of modern fuzzy set theory with renewed clarity and purpose. This journey not only enriches our understanding of uncertainty modeling but also opens new avenues for innovation in fields reliant on fuzzy logic.

Bernard De Baets is a Senior Full Professor and Head of the Department of Data Analysis and Mathematical Modelling at the Faculty of Bioscience Engineering, Ghent University. He leads the KERMIT research unit, focusing on knowledge-based systems. With a background in mathematics, computer science, and knowledge engineering, his research spans knowledge-based, predictive, and spatio-temporal modeling, with applications in the biological sciences. He has authored over 700 peer-reviewed journal papers, earning numerous best paper awards and accumulating nearly 37,000 citations.

He serves as Editor-in-Chief of Fuzzy Sets and Systems and sits on the editorial boards of several journals. He is an Honorary Professor or Visiting Professor at institutions in Hungary, Finland, Cuba, South Africa and India.

AI for Good: Machine Learning in Medicine and Healthcare – Basics, Use Cases and Prospects

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Medicine is fundamentally an empirical science: most medical knowledge arises from the careful interpretation of observational data. It is therefore not surprising that machine learning—i.e., learning from data using statistical and computational methods—has become a standard tool in medical research, particularly as medical data become increasingly large, complex, and multi-dimensional.

In many cases, medical data originates from clinical studies, in which data are deliberately generated according to carefully designed protocols. However, designing and conducting such studies is tedious, time-consuming, and expensive. At the same time, vast amounts of data are routinely collected in clinical practice, primarily for documentation purposes.

This talk aims to demonstrate the value of machine learning in deriving medical knowledge from data. In the first part, we present a case study illustrating how machine learning can be used to create a valuable clinical tool from routinely collected—yet typically underutilized—clinical data. The case study focuses on identifying a risk model for elective heart valve surgery. We show that, using state-of-the-art machine learning methods, it is possible to train risk models that outperform established clinical risk scores by leveraging past patient outcomes.

The second part of the talk introduces a new machine learning–based protocol for analyzing medical data, designed for both research and clinical use. Traditional studies often require carefully matched groups to avoid confounding, which is not always feasible in real-world settings. Our approach extends classical methods, enabling valid comparisons without perfect stratification. This expands the scope of study designs and supports meaningful analysis of routine clinical data. The talk outlines the goals and current progress of this ongoing research.

Ulrich Bodenhofer is a Professor of Artificial Intelligence at the University of Applied Sciences Upper Austria (Hagenberg, Austria). He obtained master's and PhD degrees in Applied Mathematics from Johannes Kepler University Linz and has been active in artificial intelligence and machine learning research since the 1990s. He was a long-term board member of the European Society for Fuzzy Logic and Technology (EUSFLAT), serving as its President from 2005 to 2009.

His research contributions span fuzzy relations, fuzzy rule-based machine learning, support vector machines, and, more recently, deep learning. Since June 2018, he has also served as Chief Artificial Intelligence Officer at QUOMATIC.AI, an AI startup. The application areas of his research include industrial systems, sales analytics, healthcare, and bioinformatics.

Mathematical Ideas in Quantum Algorithms

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In my talk, I will show how a range of mathematical concepts underpin the field of quantum computing. I will begin by introducing the mathematical framework of quantum information, highlighting how it distinguishes between classical randomness and quantum superposition. From there, I will present some of the key results in quantum algorithms and explore the mathematical ideas that drive them. In particular, I will discuss quantum walks as the quantum analogue of classical random walks, emphasizing the mathematical connections between the two.

Andris Ambainis is a Professor of Computer Science at the University of Latvia and a globally recognized expert in quantum computing. He earned his Ph.D. from the University of California, Berkeley, and is known for foundational contributions to quantum algorithms, including quantum walks and the quantum adversary method. His achievements have been honored with a Sloan Fellowship and an invitation to speak at the 2018 International Congress of Mathematicians.

He leads the Center for Quantum Computer Science at the University of Latvia and coordinates several international research initiatives, including the QuantERA-funded Hybrid Quantum Classical Computation project. As a co-leader of the Latvian Quantum Technologies Initiative, he is at the forefront of advancing quantum research and innovation in the region.

Part II
Advancements and Applications of Fuzzy Theory and Fuzzy Control

A New Adaptive Dynamic Window Method Combined with Fuzzy Neural Network in Complex Dynamic Environment

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Special Section: Advancements and Applications of Fuzzy Theory and Fuzzy Control
Track: Abstracts

Abstract. The traditional Dynamic Window Approach (DWA) with constant weight values of the evaluation function leads to the inability of Automated Guided Vehicles (AGV) to perform obstacle avoidance and path planning in complex environments. This paper proposes an adaptive DWA (ADWA), introducing neural network training on the basis of the Mamdani DWA (MDWA) Experiments conducted in the MATLAB simulation environment demonstrate that the improved DWA significantly enhances AGV's obstacle avoidance capabilities, optimizing trajectory and reducing travel time.

Keywords: Fuzzy control; Dynamic window approach; Neural network; Automatic guided vehicle.

Applications of Kolmogorov Arnold Networks

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Special Section: Advancements and Applications of Fuzzy Theory and Fuzzy Control
Track: Abstracts

Abstract. Multi-Layer Perceptrons (MLP) were proposed a long time ago and have now been widely used. However, as MLP has matured, its potential for further improvement has reached its limit. Kolmogorov Arnold Networks (KAN), as a promising alternative to MLP, offer better interpretability and visualization, making them advantageous in small-scale artificial intelligence applications. In this study, we utilize KAN and KAN convolution for feature extraction in robotics and employ reinforcement learning for obstacle avoidance.

Keywords: Kolmogorov Arnold Networks; Convolutional KANs; Reinforcement learning.

Dynamic Simplified Decomposed Fuzzy Systems and Its Modeling Effects Analysis

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Special Section: Advancements and Applications of Fuzzy Theory and Fuzzy Control
Track: Abstracts

Abstract. In this study, a novel dynamic structure with the use of dynamic membership functions in the so-called dynamic simplified decomposed fuzzy systems (dynamic SDFS) is proposed. It is evident that DFS possesses a very fast and good learning capability as shown in the literature. Thus, in our study, we further explored and analyzed the use of DFS with different variances. The dynamic SDFS is to act as a fuzzy approximator in adaptive fuzzy control. The proposed dynamic structure is based on DFS and SDFS to have better modeling performance with less fuzzy rules used. It is well-known that overfitting phenomena may occur in the learning process if redundant fuzzy rules are used. DFS are discovered to have overfitting phenomena when disturbances are given in the middle of learning; meanwhile SDFS does not have significant overfitting phenomena in this situation. In order to have better match in the selection of component fuzzy systems, which is a basis for simplified DFS, the proposed method considers dynamic membership functions instead of fixed antecedent-parts. The selected fuzzy sets are the most hit from DFS. The main purpose of the dynamic membership functions is to keep the useful fuzzy rules and abandon the redundant fuzzy rules. Moreover, dynamic SDFS is proposed to enhance the learning efficiency and reduce the total fuzzy rules to satisfy possible real-time constraints required in many applications.

Keywords: Adaptive robust fuzzy control; Efficient learning; Fuzzy approximator.

PSO-Based Framework for Autonomous Driving in Dynamic Environments

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Special Section: Advancements and Applications of Fuzzy Theory and Fuzzy Control
Track: Abstracts

Abstract. This research proposes an enhanced Particle Swarm Optimization (PSO)-based framework for autonomous driving in dynamic environments. The framework integrates interval-valued data representation and fuzzy control to address challenges in dynamic path planning, collision avoidance, and environmental uncertainties. By optimizing both computational efficiency and navigation accuracy, the proposed method enables reliable decision-making in real-world scenarios. The proposed framework consists of three main components:

1. PSO-Based Path Planning: Optimizes the navigation path by balancing safety, efficiency, and smoothness.
2. Interval-Valued Representation: Enhances robustness by capturing uncertainties in sensor data, such as LiDAR measurements.
3. Fuzzy Control: Provides adaptive decision-making for collision avoidance and path refinement in dynamic scenarios.

Keywords: Autonomous driving; Particle swarm optimization; Path planning; Fuzzy control; Interval representation.

Dimension Reduction Methods for Regression with Interval-Valued Data

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Track: Abstracts

Abstract. Recently, a novel regression-based clustering algorithm, known as interval fuzzy c-bivariate regression models with Box-Cox transformation (iFCBRMbc), has been proposed for the interval-valued data. This approach partitions an overall linear interval-valued regression model into multiple linear sub-models, thereby facilitating the estimation of interval-valued regression coefficients specific to each sub-model. By integrating the concept of bivariate linear regression, iFCBRMbc enhances the predictive performance and interpretability of estimated linear interval-valued regression models, thus offering a more refined approach to handling the inherent variability of interval-valued observations in this study. Despite these advantages, incorporating an excessive number of variables within a bivariate linear regression framework can lead to significant computational overhead, increased model complexity, and potential overfitting. To mitigate these issues, the present study explores the incorporation of various dimension reduction methods for regression—namely, principal components regression (PCR), partial least squares regression (PLSR), and partial least squares envelope regression (PLSER). These dimension reduction techniques aim to preserve the critical information within the dataset while filtering out redundant or noise-laden variables. Finally, the effectiveness and practical utility of the PCR, PLSR, and PLSER approaches will be thoroughly assessed using real interval-valued datasets. Empirical evaluations will focus on model accuracy, computational efficiency, and robustness, providing insightful guidelines for researchers and practitioners seeking to optimize regression-based clustering algorithms in scenarios with high-dimensional interval-valued data.

Keywords: Dimension reduction; Partial least squares envelope regression; Interval-valued data.

Designing an LSTM-Based Action Recognition System with Limited Data

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Special Section: Advancements and Applications of Fuzzy Theory and Fuzzy Control
Track: Abstracts

Abstract. This study introduces a modified approach for human action recognition using skeletal data with minimal training samples. Traditional methods often rely on extensive preprocessing, large datasets, and specialized hardware, limiting their scalability. To address this, we combine a two-stream Long Short-Term Memory (LSTM) network with a Skeleton View-Invariant Transform to effectively capture both spatial and temporal features of human motion, all from image-based inputs. Our framework optimizes action recognition performance even in scenarios with limited data, achieving high accuracy without the need for wearable sensors. Experimental results demonstrate that the proposed model delivers comparable performance to existing methods, using significantly fewer training samples. This approach not only improves human posture recognition but also offers a scalable solution for real-time motion analysis, with potential applications in gesture tracking and interactive systems, where data availability may be constrained.

Keywords: Human action recognition; LSTM; Skeletal data.

Energy-Saving Technology with Distributed Intuitionistic Fuzzy Sets for Internet of Things Systems

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Special Section: Advancements and Applications of Fuzzy Theory and Fuzzy Control
Track: Abstracts

Abstract. The rapid growth of Internet of Things (IoT) systems has led to an exponential increase in sensing devices, significantly raising power consumption. Designing efficient energy-saving control technology for IoT systems is a crucial research area aimed at reducing carbon emissions. In environments where the sensing device cannot easily access power and the battery cannot be replaced, it is essential to efficiently use the limited battery power to sustain the sensing task. Sensing devices consume a significant portion of their energy when transmitting and receiving data through wireless communication. Therefore, reducing redundant data transmission and reception is an effective method to conserve the power and energy consumption of sensing devices. To extend the battery life of sensing devices, it is imperative to design and develop efficient energy-saving technology for IoT systems. We consider a sensing event trigger-oriented application scenario to analyze and explore the routing architecture of data transmission by sensing devices, the deployment location and sensing range of these devices, and the relationship between the battery power usage time and the location of the sensing event. We propose a distributed intuitionistic fuzzy set-based energy-saving control scheme that considers factors such as remaining battery power, transmission distance, and the density of sensing devices within a given range. By intelligently selecting the most suitable devices for data transmission, this approach minimizes redundant communication. Through simulations and implementation, we validate the effectiveness of this strategy in reducing data transmission and reception. As a result, most sensing devices enter sleep mode when not actively transmitting, significantly lowering power consumption, conserving energy, reducing carbon emissions, and extending the operational lifespan of sensing devices.

Keywords: Energy-saving; Internet of Things (IoT); Intuitionistic fuzzy set; Redundant data; Sensing device.

Study on the Explainable AI of Temporal Convolutional Network

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Special Section: Advancements and Applications of Fuzzy Theory and Fuzzy Control
Track: Abstracts

Abstract. This paper proposes some methods for explainable Temporal Convolutional Network (TCN). In order to achieve explainability of TCN, this paper explains the composition of the model structure and the impact of the structure on the model itself. That is, this paper aims to enhance the explainability of TCN by adjusting the hyperparameters. In general, TCN is considered a black box neural network, making it difficult to fully comprehend its computational processes and changes. However, by understanding the impact of each neural network layer within TCN, we can achieve explainability through hyperparameter adjustment. Additionally, two explainable tools, Local Interpretable Model-Agnostic Explanations (LIME) and SHapley Additive exPlanations (SHAP), are utilized to assist in understanding the influence of data on TCN. In explainability of TCN on model structure, from the intricate examination of the one-dimensional convolution process, it becomes evident how this mathematical operation takes shape. One-dimensional convolution, as its name suggests, offers a linear approach to data processing, presenting an intuitive and methodological application in data analysis. Central to the understanding of this convolution process is the recognition of how it is deeply influenced by specific parameters. Two prominent factors stand out: the size of the kernel and the dilation rate. These are not mere variables but pivotal determinants that significantly modulate the input data channeled into the TCN. As these parameters change, so does the behavior and efficiency of the convolution, suggesting a delicate balance

that requires careful optimization. Besides, when we explore explainability tools like LIME and SHAP, we gain valuable insights into the underlying importance of features within a dataset. Such tools act as lenses, magnifying the significance and relevance of each feature. With a clearer understanding of the roles these features play, we're empowered to make informed decisions. We can strategically modify the dataset for optimal outcomes—be it through the removal of redundant features or the elimination of those that might impede or skew the training results. When we delve into the convolution mechanics specific to TCN under the explainability of TCN on model structure, it becomes possible to unravel the intricacies of hyperparameters and their consequential effects on training. A particularly enlightening tool in this pursuit is the receptive field formula. It reveals a fascinating interplay between the kernel size and dilation parameters. This formula serves as a testament to how these parameters, even under similar receptive field values, employ distinct ordering techniques. Empirical evidence further enriches this understanding. It has been observed that when the receptive field remains relatively constant, an augmentation in the kernel size tends to yield superior results.

Keywords: Explainability of TCN; Model structure; Hyperparameter adjustment; Receptive field.

Enhanced AML Transaction Monitoring via Fuzzy Equivalence in Rule-Based Systems

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Special Section: Advancements and Applications of Fuzzy Theory and Fuzzy Control
Track: Papers

Abstract. This paper introduces a novel, parsimoniously designed fuzzy logic architecture for Anti-Money Laundering (AML) transaction monitoring. In contrast to traditional Boolean rule-based systems, our approach employs a minimal yet sufficient fuzzy rule design framework that integrates essential components—flexible membership functions, semantic variable transformation, and statistical test results—to capture the inherent uncertainties in financial transactions. By leveraging a proprietary synthetic dataset that closely mirrors real banking behaviors, our model demonstrates significant improvements in recall, while mitigating vulnerabilities such as threshold manipulation. The parsimony of our design ensures that the architecture remains simple and interpretable, proving that a lean, carefully calibrated fuzzy logic system can effectively address key weaknesses in conventional AML monitoring approaches.

Keywords: Fuzzy sets; Anti-Money Laundering; Transaction Monitoring Systems.

Environmental Semantic Mapping and Obstacle Avoidance System Using Lidar and RGB-D Images

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Special Section: Advancements and Applications of Fuzzy Theory and Fuzzy Control
Track: Abstracts

Abstract. This study presents the development of an autonomous mobile robot designed for environmental semantic mapping and obstacle avoidance. The robot, measuring approximately 130 cm in height and 50 cm in width, is equipped with an RPLidar A2M12 laser scanner and an Intel RealSense D455 RGB-D camera. The laser scanner, mounted at the top of the robot, enables 2D SLAM to generate a 2D grid map of the environment. Simultaneously, the RGB-D camera, positioned 100 cm above ground level, performs real-time object recognition using YOLOv7 and Mask RCNN, projecting object sizes and distances onto the 2D grid map to construct a semantic map. Obstacle avoidance is implemented using a fuzzy rule-based approach. Objects are categorized

into three distance zones: near (<1m), medium (1–3m), and far (>3m), with appropriate navigation responses such as halting, adjusting path, or re-routing using the A* algorithm. Additionally, obstacles within specific angular zones (0°–60°, 60°–120°, 120°–180°) dictate differential wheel speed adjustments. The fuzzy rule-based system follows predefined logic: if an obstacle is detected within 1m, the robot stops or makes a sharp turn; if within 1–3m, it slows down and adjusts direction; if beyond 3m, it maintains its course but monitors for changes. The robot tracks moving objects by detecting pixel position changes to predict movement direction and velocity, facilitating dynamic obstacle avoidance. Experimental results demonstrate the robot's ability to generate precise 2D semantic maps, capturing room layouts and object locations. In both static and dynamic obstacle environments, the system effectively avoided obstacles while maintaining safe navigation and completing patrol tasks. The proposed approach contributes to advancements in environmental safety management and autonomous surveillance, enabling reliable, round-the-clock monitoring.

Keywords: RGB-D; Object recognition; Semantic map; Fuzzy rule; Obstacle avoidance.

On Approximation Properties of Stochastic Higher Degree F-Transform

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Special Section: Advancements and Applications of Fuzzy Theory and Fuzzy Control
Track: Abstracts

Abstract. The multidimensional fuzzy transform of higher degree (F-transform) has proven to be a powerful tool in data analysis and signal processing, particularly effective in high-frequency filtering and noise reduction, as evidenced in time series applications. These successes have sparked further investigation into extending the F-transform framework to stochastic processes. Initial approaches defined a stochastic F-transform heuristically, replacing the classical Riemann integral with the mean square Riemann integral. To formalize this direction, we introduce weighted Bochner spaces of random processes and define the stochastic F-transform via orthogonal projections onto subspaces of random polynomials. For a random process that is Bochner integrable on each compact subset of its domain (i.e., locally integrable), the F-transform yields a system of random polynomials. The original process can then be reconstructed through a linear-like combination of these polynomials and weighted functions that provide a fuzzy partition of the function domain. This talk explores the approximation properties of the stochastic F-transform. We will show that under specific conditions on random processes, these processes converge poinwise to the original processes. Moreover, for processes meeting specific regularity conditions, we demonstrate that the reconstructions grounded in various regularization methodologies also converge in the norm associated with the corresponding Bochner space.

Keywords: Stochastic F-transform; Weighted Bochner spaces; Random processes; Regularization methodologie.

Interval Generalized Kernel-Based Fuzzy-Possibilistic C-Means Clustering Algorithm

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Special Section: Advancements and Applications of Fuzzy Theory and Fuzzy Control
Track: Abstracts

Abstract. Interval data provides an effective means to represent data uncertainty, closely reflecting the nature of real-world datasets. Consequently, it has become a significant representation method in fields like economics and finance, biomedical sciences, and fuzzy control. Interval fuzzy clustering techniques group data based on the similarity between interval data points, thereby uncovering latent patterns within the dataset. Offering

enhanced flexibility compared to traditional fuzzy clustering, interval fuzzy clustering proves particularly useful for tasks such as image segmentation of interval data and interval-based data analysis, finding applications in areas like disease diagnosis and business analysis. This paper contributes by proposing a novel interval general kernel-based fuzzy-possibilistic C-means (IGKFPCM) clustering method specifically designed for interval data. The central idea of this method is the introduction of adjustable coefficients into both the possibilistic and fuzzy terms of the objective function. This modification provides greater control over their respective impacts, making IGKFPCM a more generalized approach than the existing interval kernel-based fuzzy-possibilistic C-means (IKFPCM). The objective function of IGKFPCM is derived using the Lagrange multiplier method, yielding iterative update rules for membership, typicality, and prototypes, which are then used for clustering. Empirical evaluations demonstrate that IGKFPCM outperforms IKFPCM and interval kernel fuzzy C-means (IKFCM) when dealing with interval data contaminated with outliers.

Keywords: Interval-value data; Kernel-based method; Generalized fuzzy-possibilistic C-means.

Part III
EUSFLAT2025 Main Track

Functors from Fuzzy Structures Categories into Category of Fuzzy Topological Spaces with Continuous Fuzzy Relations

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Special Section: EUSFLAT2025 Main Track
Track: Papers

Abstract. This paper deals with categories of L-fuzzy structures such as L-fuzzy closure and interior operators, L-fuzzy pretopologies and co-pretopologies, L-fuzzy relations, or spaces with L-fuzzy partitions) and transformations of these categories into the category of L-fuzzy topological spaces. Unlike traditional approaches that use mappings in the category of L fuzzy topological spaces as morphisms, continuous L-fuzzy relations are used, aligning with actual trends in fuzzy set theory. Functors transforming categories of the above mentioned structures into the category of L-fuzzy topological spaces with a continuous L-fuzzy relation are presented and some properties of these functors are investigated.

Keywords: Fuzzy topology; Continuous fuzzy relation; Transformation functors.

How to Deal with High-Impact Low-Probability Events: Theoretical Explanation of the Empirically Successful Fuzzy-Like Technique

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Special Section: EUSFLAT2025 Main Track
Track: Papers

Abstract. When making decisions, it is important to take into account high-impact low-probability events. For such events, traditional probability-based approach – which considers the product of the probability p that this event happens and the probability P that a randomly selected building will be destroyed – often underestimates risks. Available data has lead to an empirical table that provides a more adequate risk estimate. Most of the entries in this table correspond to the fuzzy-like formula $min(p, P)$. This paper explains this empirical result. Specifically, it explains both the effectiveness of the min formula – and also explains deviations from this formula.

Keywords: High-impact low-probability events; Fuzzy-like formulas; Hurwicz optimism-pessimism approach.

An Enhanced Multi Criteria Decision Making Model for Delivery Locker Placement Using TOPSIS and Einstein Operators in a Pythagorean Fuzzy Framework

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Special Section: EUSFLAT2025 Main Track
Track: Papers

Abstract. Multi Attribute Decision Making (MADM) is vital for evaluating diverse criteria in real-world scenarios. With the rise of e-commerce, efficient delivery systems are essential, particularly in urban areas facing demand for sustainable logistics. Delivery lockers streamline last-mile deliveries, reduce traffic congestion, and minimize environmental impact. This study proposes a robust MADM model to optimize locker placement using the TOPSIS method with Einstein operators in a Pythagorean fuzzy context. Criteria such as proximity, accessibility, costs, security, and scalability are assessed. Expert judgments are aggregated via Einstein operators to enhance decision stability. The Pythagorean fuzzy TOPSIS approach ranks locations by closeness to the ideal solution. This model offers a scalable, reliable framework for optimizing locker placements, supporting operational efficiency and sustainability in urban areas.

Keywords: Pythagorean fuzzy sets; Einstein operators; TOPSIS approach.

Direct Compatible Copulas

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Special Section: EUSFLAT2025 Main Track
Track: Abstracts

Abstract. The introduction of copulas has greatly contributed to widen the field of possible stochastic models. For this reason, in the last years, several investigations have been carried out on how to build new families of copulas and the study of their properties. While there is extensive literature on new bivariate copulas, the same cannot be said for multivariate ones, i.e. copulas with dimension greater than 2. A profound, general method is known as the distortion of a copula C by means of any function h belonging to the space E, where $E = \{h\colon [a,1] \to [0,1] \,|\, a \in [0,1[\text{ and } h \text{ is a strictly increasing bijection}\}$. When the distortion of a copula C through h is still a copula, we say that h is C-preserving. Remark that Archimedean copulas are nothing but distortions of the product: in this case, any product-preserving function is referred to as a multiplicative generator. Oddly enough, the method of building multivariate copulas through a suitable composition of copulas has been almost completely ignored in the literature, with one only exception (as far as we know). In a section of a paper by J. J. Quesada-Molina et al. (1994), a bivariate copula C is said directly comptible with a bivariate copula T when their composition $(u,v,w) \to T(C(u,v),w)$ is still a copula. In our work, we generalize such notion by allowing C to be a copula of arbitrary dimension. We extend some results when T or C are particular cases, such as the Frechet- Hoeffding bounds. Most of all, we establish both necessary and sufficient conditions for the direct compatibility of C with T. Quite surprisingly, we will show that, provided that C fulfills a mild assumption, C is directly compatible with T if, and only if, any function belonging to a suitable subclass of E, exclusively depending on the derivative of T with respect to the second variable, is C-preserving. Based upon these results, among other things we achieve a complete characterization of the class of bivariate copulas T such that the product is directly compatible with T.

Keywords: Copula; Direct compatibility; Distortion of copulas.

Robust Decisions: Bridging the Quantitative-Qualitative Gap

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Special Section: EUSFLAT2025 Main Track
Track: Papers

Abstract. When it comes to model uncertainty and make robust decision in the form of partial, set-valued recommendations, one can find a large literature on the topic for quantitative models but much less for qualitative models. This paper intends to partially solve this issue by taking notions from the quantitative world and transfer them to the qualitative one. In particular, we consider the transfer of decision rules based on lower probabilities and Choquet integral to decision rules based on fuzzy measures and Sugeno integral.

Keywords: Uncertainty; Decision making; Qualitative model; Sugeno integral; Fuzzy measures.

How to Share a Success, How to Share a Crisis, and How All This Is Related to Fuzzy

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Special Section: EUSFLAT2025 Main Track
Track: Papers

Abstract. In many practical situations, a group of people needs to share a success. What is the fair way to share this success? Nobelist John Nash showed that under reasonable conditions, the group should select the alternative for which the product of utility gains is the largest possible. This solution makes perfect sense from the fuzzy-formalized commonsense viewpoint: it maximizes the degree of confidence that all participants are happy. A natural question is: can we extend this result to a different class of situations, when a group of people needs to share sacrifices caused by a crisis? In this paper, we prove that in this case, no solution satisfies the same set of conditions. We also explain how to actually fairly distribute needed sacrifices in the case of a crisis.

Keywords: Fair division; Fair distribution of sacrifices; Nash's bargaining solution; Fuzzy techniques.

Some Results on Fuzzy Basis of Fuzzy Lie Algebras

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Special Section: EUSFLAT2025 Main Track
Track: Papers

Abstract. This work explores the application of fuzzy set theory to the study of algebraic structures, with a particular focus on Lie algebras. Since fuzzy Lie algebra can be regarded as fuzzy vector space, the foundational properties of these structures are investigated, with emphasis placed on the role of fuzzy basis. A selection of key results is presented, establishing necessary and sufficient conditions for a basis to be fuzzy, and providing concrete examples to illustrate how these findings can be applied in practice to fuzzy Lie subalgebras.

Keywords: Fuzzy set theory; Lie algebra; Fuzzy Lie algebra; Fuzzy basis; Level set.

About T-Norms and T-Conorms on New Preorders in Type-2 Fuzzy Sets

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Special Section: EUSFLAT2025 Main Track
Track: Papers

Abstract. We propose two families of operators which could be used as triangular norms or conorms in the framework of type-2 fuzzy sets. We study their properties and present two new preorders for the set of functions from $[0, 1]$ to $[0, 1]$. We show that the proposed operators satisfy the definition of t-norm and t-conorm, respectively, with the given preorders.

Keywords: Convex function; Normal function; Type-2 fuzzy set; Preorder; Triangular norm; Triangular conorm.

A Decision-Making Framework Based on Intersection and Similarity Measures for Type-2 Fuzzy Sets

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Special Section: EUSFLAT2025 Main Track
Track: Papers

Abstract. This paper introduces a decision-making framework that employs Type-2 Fuzzy Sets (T2FS), focusing on normal and convex membership functions. Decisions are constructed using the intersection of T2FS representing constraints and objectives, and alternatives are ranked based on their similarity to the ideal decision using tailored similarity measures. A case study on hotel selection demonstrates the practical utility and robustness of the framework.

Keywords: Type-2 fuzzy sets; Decision-making; Similarity measures; Selecting best cities.

On the Existence of Non-Trivial Monometrics on Betweenness Relations: Some Sufficient Conditions

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Special Section: EUSFLAT2025 Main Track
Track: Papers

Abstract. In the recent past, a special type of distance function, namely monometrics, has garnered a lot of attention because of its value and interest in both theory and applications. The key challenge lies in finding a monometric w.r.t. an arbitrary betweenness relation. To address this, the constructions of non-trivial monometrics on various types of betweenness relations have been explored. While it was shown that a non-trivial monometric may not always exist on arbitrary betweenness relations, the study of such betweenness relations has not been undertaken. In this paper, we take a step towards filling this gap by partially characterizing betweenness relations that will not give rise to non-trivial monometrics. By addressing this fundamental question, we hope to provide a clearer framework for understanding when monometrics can be defined.

Keywords: Distance function; Monometric; Betweenness relation.

A Study of the Fuzzy Differential Entropy

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Track: Papers

Abstract. Entropy is a fundamental concept in information theory but also in some AI algorithms, used for comparison of two distribution functions or measures that describe one specific event. When focusing on the differential (continuous) version of the entropy within the fuzzy measure framework, it is necessary to generalise all the essential concepts from the additive case to the fuzzy setup. This involves shifting from probability to fuzzy measures, from Lebesgue to Choquet integral and from Radon-Nikodym to Choquet-Radon-Nikodym derivatives. In the paper, two formulas for defining fuzzy entropy are proposed with the use of extended versions of the Choquet integral. Their basic properties are examined and compared with the additive case, and a relation with the fuzzy Kullback-Leibler divergence is derived. Using a novel insight into derivatives known as the resulting measure approach, some computations are presented to determine the final entropy value for two given measures.

Keywords: Fuzzy measures; Differential entropy; Choquet–Radon–Nikodym derivatives; Resulting measure approach.

Visual Comparison of Inclusion Measures

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Track: Papers

Abstract. The concept of fuzzy set inclusion is essential in decision-making, classification, and natural language processing, yet comparing different inclusion measures remains a significant challenge. To address this, we propose a visualization method using contour plots to facilitate direct comparison of different inclusion measures. By analyzing rescaled fuzzy sets, we investigate how measures behave under various parameter settings. Several well-known measures, including those based on S- and R-implications, are examined. Understanding these computational and theoretical properties is critical for selecting efficient inclusion measures in practical applications, where optimizing model performance. This study contributes to the development of systematic approaches for evaluating inclusion measures.

Keywords: Inclusion measures; Measure visualization; Measure comparison.

A Natural Extension of F-Transform to Triangular and Triangulated Domains Necessitates the Use of Triangular Membership Functions

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Track: Papers

Abstract. In many practical situations when we process 1-D data, the method of F-transform turned out to be very useful. In this method, we can use either triangular membership functions or more complex ones. Because this method has been so successful in 1-D applications, a natural idea is to extend it to functions defined on 2-D and higher-dimensional domains – e.g., to images. This method allows natural generalization to rectangular domains, where it indeed turned out to be very effective. A recent paper showed that it can extended to more general domains – e.g., to triangular domains and to more general domains that are divided into triangular domains by triangulation. Interestingly, while all 1-D membership functions can be extended to the rectangular domains, the current extension to triangular and more general domains was produced only for triangular membership functions. In this paper, we show that this restriction is not accidental: a natural extension of F-transform to triangular domains is only possible for triangular membership functions. This may explain why such membership functions are often very effective.

Keywords: F-transform; Triangular and triangulated domains; Triangular membership functions.

Locally Modified Multivariate F^m-Transform: Theoretical Background and Possible Applications

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Track: Papers

Abstract. A classical result states that internality and idempotence are equivalent for aggregation functions. In the context of data analysis, it is natural to consider random variables as the inputs of the aggregation. In this direction, aggregations of random variables are functions that, given a random vector, return a random variable fulfilling monotonicity and some boundary conditions with respect to a stochastic order. This paper is focused on the definition of different notions of idempotence and internality for aggregations of random variables. The implications between the introduced concepts are studied in detail. In addition, families of aggregations of random variables that fulfill the defined properties are provided.

Keywords: Multivariate fuzzy transform; B-spline; Extrapolation; Approximation error; Collocation method.

Prioritized Preference Aggregation for Non-Uniform Groups of Agents

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Track: Papers

Abstract. A novel approach to the important problem of aggregation of individual fuzzy preferences of a group of agents in a group decision process is presented. It is different than the traditional, widely employed aggregation based on the averaging of the respective preference degrees between pairs of options which stands for the uniformity of the group of agents in the sense that all testimonies of the agents are to be accounted for, even if the agents (and maybe also options) are assigned various importance. The novel approach proposed here assumes that the group is not uniform in the above sense which implies that the testimonies of the most important agents are crucial and if they are decisive, then they should be followed, possibly ignoring testimonies of lower level agents, as it often happens in business or the military. Yager's approach to the so called prioritized aggregation is employed. The results are very promising and can yield new vistas and perspectives for (fuzzy) preference based group decision making in complex groups of agents.

Keywords: Group decision making; Fuzzy preference; Fuzzy preference aggregation; Prioritized aggregation.

Towards Probabilistic Entropies for Interval Valued Fuzzy Sets

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Track: Papers

Abstract. Measures of information are a key concept and in the heart in Artificial Intelligence. The well-known measure of information, the Shannon entropy, is commonly used in several domains to value the information

associated with a set of probabilistic events. By extension, it is also used to provide information on a distribution of values. This aim of measuring information has been extended to the fuzzy sets theory and to its extension. Indeed, several entropies of interval-valued fuzzy sets (IVFS), or, equivalently, entropies of Atanassov intuitionistic fuzzy sets, have been proposed. However, all these entropies takes only into account the form of the set rather than a probabilistic information that could be associated to it. In this paper, we state the requirements that a probabilistic entropy of IVFS should fulfilled.

Keywords: Measure of information; Entropy; Interval-valued fuzzy sets; Intuitionistic fuzzy sets.

A First Approach to Fuzzy Logic Applied to the Explainability of Deep Learning Models by Layers

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Special Section: EUSFLAT2025 Main Track
Track: Abstracts

Abstract. The objective of this work is an initial approach to study the explainability of deep learning models at the layer level using techniques based on fuzzy logic. The fundamental idea is to associate activation levels per layer to the concepts represented by an input text to examine the coherence and relationships between similar concepts. To achieve this, a set of movie reviews is used [1], where each text is classified into one of seven emotions using a sentiment analysis model [2]. Each input text, along with its detected emotion, is associated with a vector representing the activation levels of the neural network layer. Subsequently, a fuzzy clustering algorithm [3] is applied to the obtained vectors to group them based on their values. Fuzzy clustering allows a vector to belong to multiple clusters with different degrees of membership, which is useful for capturing the ambiguous and nuanced nature of textual data. The goal is for the resulting clusters to group vectors that represent similar concepts. Finally, a detailed study of the clusters is conducted, using coherence measures to evaluate their internal consistency and entropy measures to assess their uncertainty or internal dispersion. These metrics help to better understand the structure and quality of the formed clusters, allowing the study of relationships between vectors that represent similar concepts. The preliminary results are promising and encourage us to continue this study.

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Keywords: Explainability; Deep Learning; Fuzzy Logic.

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Bipolarity on Bounded Lattices

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Special Section: EUSFLAT2025 Main Track
Track: Abstracts

Abstract. Bipolarity, and especially bipolar ordered weighted average operators on the real line attract attention in recent years. We are going to contribute to this topic by discussing bipolarity and bipolar ordered weighted average operators on bounded lattices that may contain incomparable elements. We will present some possibilities how to work in the case of incomparable elements. Roughly speaking, there are two basic possibilities. The first possibility is to use the same weights and the other possibility is to extend the lattice order (i.e., to use an admissible order). Both of these cases lead to a kind of bipolar Sugeno integral. We will provide also some examples illustrating the achieved results.

Keywords: Bipolar OWA; Bipolar Sugeno integral; Bounded lattice.

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From Lower Boundary Approach to Partial Fuzzy Logics to the Upper Boundary One and Back

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Special Section: EUSFLAT2025 Main Track
Track: Abstracts

Abstract. Three-valued logics and later on their extension to partial fuzzy logics constitute an interesting area of algebraic and logical models of undefinedness. In particular, truth values that are not defined may represent distinct types of, say N/A values we are familiar with from database systems, such as irrelevant, contradictory, genuinely undefined, or simply missing values. Technically, the algebraic counterpart of partial fuzzy logics is based on extending the algebraic support $[0, 1]$ by a dummy value * and consequently, extending the algebraic operations to operate on the extended support. It has been shown that some of such algebras, namely those representing so so-called lower boundary approaches that try to mimic the lower guaranteed estimation of the truth values, may serve well for the computational machinery in the case of modeling missing (unknown) values.

Although the design was made in order to preserve as many residuated lattice properties as possible, the genuine residuated lattice structure has not been reached. Interestingly, when an analogous approach was employed dually to come up with a sort of upper boundary strategy, the residuated lattice structure was obtained.

This success motivates this contribution that is based on introducing again the backward dual design of another lower boundary approach and the consecutive investigation of its residuated lattice properties. We show that under specific assumptions, the residuated lattice algebra may be reached.

This fact may play a crucial role in real/worldcases such as decision-making or any other knowledge systems with missing data, which is, rather usual case in social sciences.

Keywords: Partial fuzzy logic; Lower boundary algebra; Upper boundary algebra; Residuated lattice; Zero divisors.

Kernel Operators on Trellises

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Special Section: EUSFLAT2025 Main Track
Track: Abstracts

Abstract. A key property in lattice theory is the transitivity of the ordering relation or, equivalently, the associativity of the lattice operations. The absence of transitivity is often inconvenient for mathematicians and computer scientists (see the interest in acyclic directed graphs, for example). Nevertheless, it might provide other scenarios where preference loops or incomparability can be considered, such as in species competition structures.

In this work, we focus on interesting mathematical structures that are less known than lattices, namely the class of pseudo-ordered sets (psosets, for short), and in particular the subclass of trellises [H. Skala, Trellis theory. Algebra Universalis, 1 (1971), pp. 218-233], also called tournament lattices, non-associative lattices or weakly associative lattices. Recently, in [L. Zedam, B. De Baets, Triangular norms on bounded trellises. Fuzzy Sets and Systems, 462 (2023) 108468] the authors introduce the notion of a t-norm on bounded pseudo-ordered sets and in particular on bounded trellises. Another outstanding notion in fuzzy set theory is that of a kernel operator, in particular, the 'very-true' or 'hedge' operator. In this work, we address the study of kernel operators and their counterparts, closure operators, within the general framework of psosets and trellises. Furthermore, we also explore Galois connections, which are closely linked to these concepts.

Keywords: Trellises; Kernel operator; Galois connection.

Multi-Metric Genetic Optimization of Ordinal Sums of Conjunctive and Disjunctive Functions in Classification Tasks

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Special Section: EUSFLAT2025 Main Track
Track: Abstracts

Abstract. Ordinal sums of conjunctive and disjunctive functions are suitable for handling uncertainty and the inclination toward binary classes [1]. The work [2] has shown benefits in applying ordinal sums in edge detection [3]. This work proposes a complex learning method for ordinal sums parameters [4] using genetic optimization to improve the results, the classification interpretability, and the explainability.

We compared the results obtained on the California housing dataset with logistic regression, random forest, and Naive Bayes. The model using ordinal sums achieved a root mean square error of 0.1613, outperforming logistic regression (0.3802), Naive Bayes (0.4262), and random forest (0.4439). This result underscores the advantage of ordinal sums in capturing the nuanced inclination of records towards certain classes.

When evaluating the classification accuracy, the ordinal sums model achieved an accuracy of 0.8708, compared to 0.8554 for logistic regression, 0.8030 for random forest, and 0.8184 for Naive Bayes. Similarly, the F1 score for the ordinal sums model was 0.7491, closely aligned with logistic regression (0.7897) and Naive Bayes (0.7510), and outperforming random forest (0.6561). These outcomes demonstrate that, while performance is comparable in certain scenarios, the ability of ordinal sums to model inclination and handle asymmetric classes leads to more consistent and interpretable classification.

Future research should focus on a hybrid fitness function in genetic algorithms, which might enhance performance across varying datasets and tasks. The next direction is extending the framework to handle multi-class problems.

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Keywords: Ordinal sums; Classification; Genetic algorithms; Agregation functions Explainabilty.

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A Fuzzy Logic Framework for Decision-Making Support Using a Variant of TSK Models

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Special Section: EUSFLAT2025 Main Track
Track: Abstracts

Abstract. This work aims to address the automatic generation of fuzzy decision models based on expert knowledge. To achieve this, a variant of the Takagi-Sugeno-Kang Fuzzy Inference System (TSK FIS) and a framework capable of generating decision models through a set of parameters defined by experts are proposed. This framework introduces a variable co-occurrence mechanism that captures interactions, allowing experts to intuitively define relationships between variables. This mechanism ensures that the expert does not need a deep understanding of fuzzy logic. To ensure everything functions correctly, the inference mechanism has also been modified to allow decision rules to produce robust and interpretable results by obtaining the output value cumulatively.

To test the proposal, it has been applied to a problem of weapon selection in video games by players according to their profile. Experimental results have validated the system's ability to generate decision models that adapt to users' specific profiles and objectives, maintaining both accuracy and interpretability in decision-making.

Depending on the number of variables and their co-occurrence, the mechanism can generate a large number of rules. To mitigate this effect, future work will focus on simplifying the model by merging rules and reducing variables, updating the consequents in the process while preserving accuracy, robustness, and clarity. Additionally, when datasets are available, a fully automatic induction method for these decision models will be developed to adjust the parameters based on an input data set.

NOTE: This abstract describes the work 'A Knowledge-Driven Fuzzy Logic Framework for Supporting Decision-Making Entities' which is being submitted to the journal 'Applied Soft Computing'. It is currently in the 'Minor Revision' stage pending very minor modifications. We are confident that it will be published by the time of the conference.

Keywords: KD-DSS; Fuzzy rules; Decision Models; TSK FIS.

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Can Fuzzy Fingerprints Really Make LLMs More Interpretable?

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Special Section: EUSFLAT2025 Main Track
Track: Abstracts

Abstract. The concept of Fuzzy Fingerprints (FFP) was introduced by Homem and Carvalho in 2011, and has been successfully applied to various text classification tasks, including authorship detection in news articles and tweets, identifying topics and detecting cyberbullying in social networks or classification using medical notes and reports. A FFP is a fuzzy set on the discrete universe of the features used in the problem definition.

The essence of the original FFPs lies in their ability, for any text classification dataset, to capture the identity of a given class through feature frequency, thereby minimizing the probability of collision, i.e., two classes yielding the same fingerprint. The authors achieve this by considering the set of texts associated with a given class and using the frequency of each word in each text to construct the class FFP. For all classes they obtain a FFP library. To classify an unseen text, a FFP is generated for the text using the same method employed to build the FFP of a class. The newly created fingerprint is then compared with the fingerprints in the library to identify the most similar match, thus determining the appropriate class for the text, while allowing for interpretable results. The similarity is based on the aggregation of the fuzzy intersection of the FFPs (each of which is a fuzzy set). The interpretability of the process arises from the fact that this is essentially a prototype-based classification method, where the distance of an instance to be classified to each class prototype is clearly defined and readable due to its fuzzy nature.

FFP were very successful in text identification and classification, especially in tasks with a large number of classes, but the introduction of the self-attention mechanism and Transformer Models which started taking full advantage of the concept of embeddings as features in NLP tasks, led to a revolution where models such as Bidirectional Encoder Representations from Transformers (BERT), or RoBERTa, soon became state-of-the-art in all text classification tasks, largely outperforming models such as the above mentioned FFP. These models soon started to be known as Large Language Models (LLM).

Despite having a huge performance advantage in classification tasks (and many other advantages), LLM are still massively complex black boxes that lack interpretability and explaining the obtained outcomes from these models is still a challenge, since they can fail catastrophically in situations of apparent simplicity and hardly anyone can understand or explain the reasons for such specific fail. These failures became colloquially known as "hallucinations".

FFP cannot compete in classification performance with LLMs but are able to provide a way larger degree of interpretability and explainability in classification tasks. In 2022 a framework was successfully proposed to combine FFP and the BERT classification layer to obtain lighter and potentially more interpretable LLM classification models. The proposed implementation showed that it was possible to largely reduce the number of neurons in the LLM output layer without losing classification performance, but the interpretability of the model was not consensual. Here we investigate how, why and to what extent the process of Fuzzy-fingerprinting a LLM can make the classification process interpretable.

Keywords: Fuzzy Fingerprints; Large Language Models (LLM); Classification; Interpretability.

Fuzzy Clustering-Based Aggregation for Credit Risk Assessment

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Special Section: EUSFLAT2025 Main Track
Track: Abstracts

Abstract. Our research deals with the special construction of aggregation, which is based on fuzzy clustering process. In decision making there can be the need to aggregate values, which represent evaluations provided by several experts. In such cases aggregation operators based on similarities could be used. In the context of risk management we previously proposed an approach when similarity between objects under evaluation is established by applying fuzzy clustering and is used to aggregate estimates of the risk levels obtained from the experts.

Suppose we have the initial data set (learning set) of customers with different attributes, which somehow characterize these customers. The experts have estimated credit risk levels for these customers, by using their own analytical approach and skills and obtained an evaluation of each object. The set of customers is divided between fuzzy clusters, taking into account the attributes of these customers. General aggregation operators could be used in order to obtain a credit risk level estimation for each cluster. Then, a credit risk rating could be assigned for a new customer by determining it's membership degree to each established cluster.

In this talk we study the application of different fuzzy clustering methods and corresponding parameters of these methods, different T-norms and ordinary aggregation operators, and compare results from this perspective.

Keywords: Fuzzy clustering; Aggregation operator; Risk management; Decision making.

Uniqueness and Reconstruction of Strict T-Norms Known Only Above a Level Curve and Different Curves

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Special Section: EUSFLAT2025 Main Track
Track: Abstracts

Abstract. Investigating how to extend t-norms defined on a subset of the unit square (referred to as pre-t-norms) to the entire domain is a fundamental and long-standing challenge in the research of these operators [1]. Usually, the problem is studied under continuity and Archimedean assumptions, because the existence of additive/multiplicative generators reduces the problem to the study of systems of functional equations which depend on the known region. Although there exists plenty of results dealing with different regions, only in a few cases the explicit construction of the completion is provided and/or the unicity is ensured.

Recent characterizations of (S,N)-implications encouraged the study and resolution of the completion problem for new subregions [2]. Interestingly, those results are rooted in a novel result that ensures the construction and uniqueness of the additive generator of a continuous and cancellative pre-t-norm known above a level curve. Although this result was used with the main purpose of solving the particular problem in [2], it has been pointed out that it may be useful to understand further which regions of the unit square contain enough information to reconstruct a t-norm.

In our work, we are interested in applying this result to study which additional information should we add to the region above a level curve to ensure a unique completion of a continuous and cancellative pre-t-norm. In particular, we consider the diagonal, a vertical/horizontal section and, more generally, an arbitrary curve. For all the considered cases, we confirm the existence and uniqueness of the completion and we provide its construction in terms of the additive generator.

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Keywords: T-norm; Pre-t-norm; Continuous completion.

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On Combination of T-Conorms for Aggregation of Data in Risk Assessments

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Special Section: EUSFLAT2025 Main Track
Track: Abstracts

Abstract. Aggregation is widely used for data analysis and obtaining meaningful results illustrating the final outcome of different research. Risk assessment processes play a key role in various industries. We base our studies on examples relevant for financial industry where results of the risk assessments are important for such areas as servicing clients, granting credits and assessing financial stability of different entities. As part of single aggregation we apply the most common t-norms and t-conorms for consolidation of data. While t-norms are traditionally used for such purposes, we demonstrate how they differ from application of t-conorms. In several cases it is evident that t-conorms are more suitable for aggregation of risk related data, e.g. risk levels. T-conorms can be also multiplied by fuzzy coefficients to characterize importance of relevant risk parameters. Some of t-conorms can be combined and aggregated for obtaining more meaningful results. We explore embedding of Łukasiewicz t-conorm into maximum t-conorm and applying arithmetic average of product t-conorm and maximum t-conorm.

Keywords: Aggregation; T-norms; T-conorms; Risk assessment.

Weighted Quantile Approach to Time Series Forecasting Using Fuzzy-Probabilistic Inference Systems

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Special Section: EUSFLAT2025 Main Track
Track: Abstracts

Abstract. Quantile-based approaches have become increasingly valuable in time series analysis and forecasting, particularly for capturing distributional dynamics, managing heavy-tailed data, and enhancing robustness in

volatile environments. These methods originate from quantile regression theory, pioneered by Koenker and Bassett [1]. A prominent example of such methods is quantile autoregression [2], which directly models conditional quantiles of the time series. Recently, weighted quantile methods have gained popularity, extending standard quantiles by incorporating data-dependent weights in their estimation. Several techniques have been proposed to derive weighted quantiles, including kernel-based approaches [3] and the Weighted Harrell–Davis quantile estimator [4]. While weighted quantiles have found practical applications in time series analysis, their use in time series forecasting remains relatively limited (see, e.g., [5]).

This talk introduces a novel time series forecasting method based on a fuzzy-probabilistic inference system. The central concept is the use of an IF–THEN rule system, where antecedents are represented by fuzzy sets forming a fuzzy partition of the time domain, and consequents are quantile functions that characterize the conditional distribution of time series values over each antecedent. The fuzzy-probabilistic framework was initially proposed in [6], where the quantile functions were estimated from data using a weighted quantile approach as described in [7]. The inference mechanism computes an aggregated quantile function as a linear combination of local quantile functions, with weights determined from the fuzzy antecedents following the principles in [7]. Forecasting proceeds in two stages:

1. Design of the initial IF–THEN rule system using historical data,
2. Extension of the rule system by generating additional rules for future time points.

In this step, the fuzzy sets in the antecedents naturally extend the fuzzy partition into the future, while the corresponding quantile functions are forecasted using autoregressive modeling of the differences between successive quantile functions. A family of autoregressive models, varying in the number of lagged differences used, is evaluated. The best-performing model is selected for each case, and the resulting forecasted quantile functions are inferred using the same fuzzy-probabilistic mechanism as in the original system. The effectiveness of the proposed method is demonstrated through both algorithmic implementation and empirical evaluation on real-world data.

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Keywords: Time series analysis and forecasting; Weighted quantiles; Quantile regression; Fuzzy-probabilistic inference system; Autoregressive models.

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Is There Something Fuzzy in Hypotheses Testing?

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Special Section: EUSFLAT2025 Main Track
Track: Abstracts

Abstract. Since their introduction back in 1965 by Zadeh, fuzzy sets have been used to model uncertainty, handle imprecise data, and interpret linguistic information characteristic of human reasoning. Over the years, the theory of fuzzy sets has been extensively developed, both in theoretical foundations and practical applications. Due to their adaptability to human behavior, fuzzy logic and fuzzy systems are now deeply embedded in everyday decision making. Also, various statistical models are highly applicable in many areas of decision-making. A question that has been attracting increasing attention from researchers for some time now is the synergy between statistical techniques and the flexibility of data modeling using fuzzy sets.

Three major research directions have emerged in this field. The first applies fuzzy set theory to crisp data. The second focuses on extending classical probabilistic and statistical concepts to fuzzy domains. The third investigates the benefits of integrating these two approaches.

This contribution focuses on the third direction by exploring the horizontal fuzzy relation to quantify the degree to which an observed value of a test statistic deviates from a fuzzy threshold defined by a critical value. In this setting, all quantities, including the observed value and the threshold, are modeled as fuzzy numbers. A comparison with other recent approaches to statistical issues in fuzzy surroundings has also been considered.

Keywords: Fuzzy values; Horizontal fuzzy relation; Hypotheses testing.

Part IV
Fuzzy Implication Functions

Implication Construction Methods on Bounded Lattices

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Special Section: Fuzzy Implication Functions
Track: Papers

Abstract. We present some construction methods to obtain fuzzy implications on a bounded lattice L via two implications dened on sub-intervals of the bounded lattice L. Then, we take this idea further to produce fuzzy implications via given three implications. Moreover, many illustrative examples are included.

Keywords: Fuzzy implication; Construction method; Sub-interval; Bounded lattice.

On the Threshold-Based Generalization of the Modus Ponens Syllogism

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Special Section: Fuzzy Implication Functions
Track: Abstracts

Abstract. The main goal of this presentation is to discuss a threshold-based generalization of the Modus Ponens syllogism. Our approach is unique in dealing with inequalities in the threshold-based Modus Ponens. As an implication of our reasoning, we use two variants of fuzzy implication functions: a) the preference implication operator family introduced by Dombi and Baczynski in 2020, b) the mean pliant S-implication operator family introduced by Dombi and Baczynski in 2021. We also show that the Modus Tollendo Tollens, Modus Ponendo Tollens, and Modus Tollendo Ponens' syllogisms can be generalized similarly.

Keywords: Fuzzy logic; Fuzzy connectives; Fuzzy implication functions; Threshold-based reasoning.

On a Generalization of the F-Chains Based Construction of Fuzzy Implication Functions

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Special Section: Fuzzy Implication Functions
Track: Abstracts

Abstract. Fuzzy implication functions are one of the fundamental operators in fuzzy logic, serving as generalizations of the classical conditional to other domains, such as the unit interval. Their definition imposes only monotonicity and boundary conditions, allowing for a broad spectrum of operators with diverse behaviours. This flexibility has led to the introduction of numerous families of fuzzy implication functions, each with distinct properties and applications. However, the increasing variety of such families underscores the need for deeper theoretical understanding [1], particularly regarding their structural relationships and classification.

In this work, we focus on construction methods which use different techniques for generating new fuzzy implication functions from existing ones. Specifically, we generalize the F-chains based construction, initially introduced to extend a construction method of aggregation functions to the case of fuzzy implication functions [2]. Our generalization is based on two main ideas. First, the use of a collection of fuzzy implication functions instead of a single and not necessarily an identical one for each variable, and second, the use of two different increasing functions satisfying some boundaries conditions instead of a unique F-chain. We analyse in depth the preservation of additional properties under this generalized construction, establishing the necessary conditions.

Moreover, we reveal that the F-chains based construction serves as a unifying framework for several existing methods. In particular, we prove that diverse construction techniques such as contrapositivisation, aggregation or vertical/horizontal threshold methods can be reformulated in terms of our generalized approach. Thus, we highlight structural similarities between seemingly distinct strategies. Consequently, the generalized F-chains based method provides a cohesive perspective on the landscape of fuzzy implication construction methods apart from being a standalone tool.

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Keywords: Aggregation function; Fuzzy implication function; Construction method; F-chain.

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Part V
Fuzzy Metric Spaces and Their Generalizations

Some Metric-Like Structures and $\oplus$-Based Semi(Pseudo)-Metric-Like

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Special Section: Fuzzy Metric Spaces and Their Generalizations
Track: Papers

Abstract. This study investigates metric-like spaces as a generalization of partial metric spaces introduced by Harandi. We examine the properties of open balls and points in these spaces, along with the behaviour of convergent sequences. Additionally, we introduce the semi(pseudo) metric-like structure by using the concept of extended t-conorms.

Keywords: Equal-like points; Extended t-conorm; Metric-like; Semi(Pseudo) Metric-like.

Fuzzy Equivalence Based Metrizable Space

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Special Section: Fuzzy Metric Spaces and Their Generalizations
Track: Papers

Abstract. In this paper we introduce and analyze the concept of T-equivalence spaces, which are defined as pairs (X, E), where X is a set and E is a T-equivalence relation on X. Our primary focus is on the structure of balls in these spaces and their topological properties. Specifically, we define balls as α-cuts of a T-equivalence and prove that thus defined balls are open sets. In the metrizable case, we investigate how these relations can be expressed through a metric and an additive generator.

Keywords: Metric; Fuzzy metric space; Fuzzy relation; Fuzzy equivalence; Topology.
Acknowledgments:
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On Fuzzy Metrics Constructed from Metrics and Their Topology

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Special Section: Fuzzy Metric Spaces and Their Generalizations
Track: Papers

Abstract. Fuzzy metrics have demonstrated to be useful in engineering problems. They show a better performance comparing with their classical counterparts. So, to obtain new examples of fuzzy metrics or methods to construct them is a topic of interest. Recently in [**?**] two new methods to construct fuzzy pseudo-metrics from pseudo-metrics have been established. The aim of this paper is to determine the relationship between the topology induced by the fuzzy pseudo-metric constructed using these two methods and the topology induced by the pseudo-metric from which is it constructed. In this direction, we show that for one of these two methods both topologies coincide whereas for the other one, two additional conditions on the one of the functions used in the method must be required to get such a conclusion. In addition, examples which justify and illustrate our study are provided in the paper.

Keywords: Fuzzy metric space; Continuous triangular norm; Induced topology; Additive generator; Superadditive function.

Part VI
Fuzzy Relations and Applications

Chatbots with Character - An implementation of Fuzzy Conversational Character Computing

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Special Section: Fuzzy Relations and Applications
Track: Papers

Abstract. Despite the advancements in conversational AI, most chatbots fail to adapt dynamically to users' emotions and personalities. This paper presents the Fuzzy Conversational Character Computing (FCCC) framework, which integrates Fuzzy Logic, Computing with Words and Perceptions, and Character Computing to enable chatbots to deliver adaptive, sentiment-sensitive responses. By leveraging Large Language Models (LLMs) and real-time sentiment analysis, FCCC fosters more empathetic and personalized interactions.

Through an experimental evaluation in the healthcare domain, we demonstrate that FCCC-enhanced chatbots positively influence user sentiment and satisfaction, outperforming traditional bots in perceived empathy and adaptability. These findings establish FCCC as a breakthrough in conversational AI, with broad potential for applications in healthcare, customer service, and beyond. Future research will focus on scaling the framework and exploring its integration with advanced AI technologies.

Keywords: Chatbot; Character computing; Large Language Model; Fuzzy logic.

A Few Notes to the Fuzzy Best-Worst Method

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Special Section: Fuzzy Relations and Applications
Track: Papers

Abstract. Multicriteria decision-making plays a key role in various research and practical fields, with one of the most crucial aspects being the accurate determination of criterion weights. Several methods exist, including the exact and approximative Saaty method, the Best-Worst Method, and its fuzzy variant called the Fuzzy Best-Worst method. The Fuzzy Best-Worst Method allows for handling uncertainty by assigning interval values to criterion importance. This paper addresses key questions regarding the correctness of assigning fuzzy numbers to linguistic terms, the consistency of fuzzy numbers in programming, and the appropriateness of criteria for weight determination. The discussion is based on the Transportation Mode Selection problem, initially studied by Jafar Rezaei, 2015, and later explored in a fuzzy environment by Guo and Zhao, 2017, and by Dong, Wan, and Chen, 2021. Their research introduced refinements in the Fuzzy Best-Worst Method, particularly using triangular fuzzy numbers to improve decision-making accuracy. Our goal is to contribute to the further development of the mentioned method and to foster discussions on its accuracy and applicability in practice.

Keywords: Fuzzy number; Fuzzy Best–Worst Method; Indicators; Weights.
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Incomplete Preference Relation Analysis for MgGDM System

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Special Section: Fuzzy Relations and Applications
Track: Papers

Abstract. Decision-making is a process inherent to everyday life and momentous situations, which is often complicated, especially in group contexts, due to the diversity of opinions and constraints in the available information. A novel multi-granular group decision-making method based on individual similarity is presented in this context. This method is also a consensus model that secures flexibility by allowing individuals to provide information in numerical format and based on their perspective, avoiding the need to fill in the reciprocal preference relationships with information they do not know. Furthermore, the system assigns weights to individuals based on the quality and quantity of their contributions, recognising and rewarding those who provide relevant and valuable information. Ultimately, this approach promotes a more reliable and accurate decision-making process, tailored to the knowledge and experience of the group of individuals involved, thus improving the quality of decisions made as a team.

Keywords: Consensus model; Reciprocal preference relations; Multi-granular method; Similarities; Group decision-making.

Generating Modular Relaxed Pseudo-Metrics by Aggregation

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Special Section: Fuzzy Relations and Applications
Track: Papers

Abstract. The aggregation of different pieces of information is a well-known practice in the field of applied sciences. Frequently, such information is obtained from metrics and, consequently, the goal is to merge a collection o metrics into a global one. In recent studies, the addition of a parameter to the distance measurement has been essential. Modular metrics completely meet this requirement. Therefore, in this paper we introduce and solve the aggregation problem for a new type of modular metric: the modular relaxed pseudo-metric. This new concept has been introduced to provide a less restrictive distance measurement that does not need to fulfill the axiom of reflexivity, with the aim of covering a wider range of applications. Thus, we characterize the functions that aggregate modular relaxed pseudo-metrics, which we call modular relaxed pseudo-metric aggregation functions. We also compare the results with the modular (pseudo-)metric case. We find that properties such as subadditivity and preservation of triangular triplets are necessary conditions for modular relaxed pseudo-metric aggregation functions, but not sufficient conditions. This shows a difference with modular (pseudo-)metric aggregation functions.

Keywords: Modular pseudo-metric; Modular relaxed pseudo-metric; Aggregation; Monotony; Subadditivity; Triangular triplet.

Prostate Cancer Diagnosis: A Geometric Approach Based on the Beer Index

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Special Section: Fuzzy Relations and Applications
Track: Papers

Abstract. Convexity is a well-known property of sets, but it is still not clear how to measure the degree of convexity of a set. In this paper, we seek to measure the degree of convexity of a set making use of the Beer index, which is based on the visibility of its points. The aforesaid index involves the so-called visibility function, a continuous function that allows to compute the area of the region that sees every point. An application of this method for measuring the convexity degree to prostate cancer diagnosis is presented. Here, pathologists aim to determine whether a patient has a potential risk of cancer. In our case, the Beer index is used to determine the state of individual glands present in biopsy samples by determining the degree of convexity of the lumens. This is done to help pathologists to assess whether these glands are healthy or exhibit a cancerous state. Thus, the main objective in this work is to show that the Beer index captures and reflects a morphological feature which is inherent in prostatic glands and, therefore it allows the distinction between healthy and pathological glands.

Keywords: Convexity; Visibility function; Beer index; Prostate cancer.

On Modular Fuzzy Equivalences, Aggregation and Modular Pseudo-Metrics

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Special Section: Fuzzy Relations and Applications
Track: Papers

Abstract. Since the introduction of the notion of fuzzy equivalence, many studies have explored theoretical aspects and its applications. In particular, two theoretical aspects have attracted the attention of many researchers. On the one hand, methods for generating from a collection of fuzzy equivalences a new one by means of aggregation have been extensively studied. On the other hand, the relation between fuzzy equivalences and pseudo-metrics has been profusely explored. Moreover, characterizations of those functions that are useful for merging a collection of fuzzy equivalences have been provided in terms of particular constructions of functions that aggregate extended pseudo-metrics in such a way that the construction takes advantage of the aforementioned duality relation. This type of characterizations are yet to be examined in the modular framework. This is why, in this paper, we focus our efforts on obtaining modular versions of the previously mentioned characterizations through the use of the duality relationship. Furthermore, we finally compare our new correspondences to the ones coming from the non modular scenario, pointing out their differences.

Keywords: Modular fuzzy equivalence; Modular pseudo-metric; Aggregation; Additive generator; Pseudo-inverse.

On the Impossibility of Universally Transforming Similarity Metrics into Partial Metrics

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Special Section: Fuzzy Relations and Applications
Track: Papers

Abstract. Partial metrics generalize traditional metrics by allowing non-zero self-distances. This distinguished property makes them suitable for the development of many applications to computer science, articial intelligence and applied mathematics. Such distances are interpreted as a dissimilarity measure. However, in cases where the measurement method must quantify the degree of common information between two objects, rather than quantifying the level of dierence between them, it is required to handle the notion of similarity. This paper studies a duality relationship between the so-called similarity metrics and partial metrics. In particular, we focus on the search of a characterization of those functions that transform every similarity metric into a partial metric. While transformations can exist for specic similarity metrics, we prove the non-existence of a function that can universally transform any similarity metric into a partial metric.

Keywords: Metric; Partial metric; Similarity metric; Duality.

On Metric Aggregation Functions and Fuzzy Decision-Making

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Special Section: Fuzzy Relations and Applications
Track: Papers

Abstract. In fuzzy decision-making it is required to aggregate numerical pieces of information that incorporate vagueness and that comes from various sources in order to get a unique numerical value that is incorporated in some decision-making technique and, hence, it allows to select one option between several available. In order to decide which alternative is the best one, an ideal profile is defined and the alternative chosen is exactly the one that minimizes the distance to the aforementioned ideal. The Ordered Weighted Averaging (OWA) is an instance of aggregation function which appears to be well-suited to be applied in these type of situations. Concretely, it has been shown to be useful in generating mean distances which are used to evaluate all different strategies and to find the best one according to the interest of the decision maker. However, in a natural way, the decision process imposes many times that the aggregate information corresponds to an overall distance between alternatives rather than an average distance. In this direction, metric aggregation functions result more useful. In this paper, we introduce a technique to generate this type of functions and we show a few limitations of the OWA for this purpose. Instances of such functions induced by the new technique are provided. Moreover, such examples are used to illustrate the usefulness of this type of functions in fuzzy decision-making. The selection of strategies are compared with those provided by the OWA.

Keywords: OWA; Metric aggregation function; Decision-making; Investment strategies.

Fuzzy Logic-Based Mathematical Structures in Classification Problem

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Special Section: Fuzzy Relations and Applications
Track: Abstracts

Abstract. This work studies the k-NN classification algorithm, where fuzzy equivalence relations are used as a measure of the similarity, which can be expressed in different ways via the distance (e.g., Euclidean, Manhattan, Hamming) between two objects: $s(x,y) = e^{-d(x,y)}, s(x,y) = \frac{1}{1+d(x,y)}$. Transitivity is defined based on various t-norms: Lukasiewicz, product, and Hamacher's t-norm. The construction of a fuzzy equivalence relation involves a tool called an additive generator. For the final decision regarding the class to which an object belongs, aggregation operators are introduced for the corresponding equivalence relations. Additionally, the work provides a comparative analysis between standard classification methods (k-NN, Logistic Regression) and the proposed method, which incorporates fuzzy equivalence relations where transitivity is defined based on t-norms.

Keywords: KNN classification; T-norms; Additive generator; Aggregation of fuzzy equivalence relations.

A Parametric Dissimilarity Measure for Pythagorean Fuzzy Sets and Its Application to Pattern Recognition

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Special Section: Fuzzy Relations and Applications
Track: Abstracts

Abstract. This paper introduces a novel parametric dissimilarity measure for Pythagorean fuzzy sets (PFSs), constructed using a generalized Jensen-type divergence. The proposed measure quantifies dissimilarity among the squared membership values, non-membership values, and hesitancy degrees of PFS elements through a tunable divergence parameter. Theoretical analysis establishes that the measure satisfies the fundamental properties of a distance function, namely non-negativity, symmetry, and the identity of indiscernibles, for all positive values of the parameter. Moreover, it is observed that the triangle inequality holds under specific conditions, enabling the measure to exhibit behavior characteristic of a true metric within those constraints. To demonstrate its practical utility, a case study on pattern recognition is presented, showcasing the measure's effectiveness in distinguishing fuzzy patterns under uncertainty. Comparative results highlight the robustness and adaptability of the proposed approach.

Keywords: Pythagorean fuzzy sets; Jensen divergence; Dissimilarity measure; Pattern recognition.

On Behavior of Fuzzy Morphological Operators Under the Morphisms from Category L-FSLin

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Special Section: Fuzzy Relations and Applications
Track: Abstracts

Abstract. The work is developed in a categorical framework of fuzzy mathematical morphology. We introduce a category whose objects are linear spaces X enriched with the fixed L-fuzzy subset S, together denoted by (X, S). Here, L is a linearly ordered complete lattice. The L-fuzzy subset S plays the same role as a structuring element in the classical fuzzy morphological space. The morphisms in this category are linear mappings from (X, S) to (Y, T), that preserve vector addition and scalar multiplication while respecting the fuzzy structuring elements S and T, in sense that the image of the element S is less or equal to the element T. The mapping of the structural element S is defined by Zadeh extension principle. This category is called L-FSLin. Within this framework, we study the behavior of fuzzy morphological operators - erosion ES, dilation DS, opening OS, and closing CS. The operators are examined under the morphisms from category L-FSLin. We aim to demonstrate that under certain conditions, mainly based on the inner connection between the structuring elements of the spaces, the morphisms "preserve" (in the sense "do not diminish") all fuzzy morphological operators.

Keywords: Linearly ordered complete lattice; fuzzy mathematical morphology; L-fuzzy morphological operators.

Part VII
Generalized Quantifiers, Logical Syllogisms and Applications

Using Intermediate Quantifiers to Reduce the Number of Linguistic Rules for Diagnosing Mood Disorders from Incomplete Data

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Special Section: Generalized Quantifiers, Logical Syllogisms and Applications
Track: Papers

Abstract. This paper aims to establish a method for detecting major depressive episodes experienced by people with bipolar disorder and major depressive disorder. The method only considers motor activity and total sleep time per day. Both symptoms were measured using actigraphy. Motor activity and total sleep time are usually evaluated subjectively according to the patient's statements or a professional's perception. However, it is better to use a linguistic description of the patient's situation based on objective measurements. As a test bed, we used a public domain database concerning actigraphy data of merely 55 participants that either have or do not have one of the two aforementioned mood disorders. We resorted to fuzzy natural logic (FNL) since this theory takes linguistic expressions and human reasoning into account and seeks logical conclusions that are applicable to complicated and real situations, thus being a resource that helps to develop diagnostic reasoning and not replace it. Initially, we created implicative rules based on already established knowledge about mood disorders and designed fuzzy sets that describe the total sleep time and motor activity of each participant in terms of evaluative linguistic expressions. Using intermediate quantifiers, we were able to reduce the number of linguistic rules from fifteen to three.

Keywords: Fuzzy natural logic; Mood disorder; Medical diagnostic support; Actigraphy; Motor activity; Sleep time.

Verification of Validity of Generalized Logical Syllogisms Applying the Contraposition

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Special Section: Generalized Quantifiers, Logical Syllogisms and Applications
Track: Papers

Abstract. This publication closely follows previous results concerning verifying the validity and invalidity of logical syllogisms with intermediate quantifiers that form selected logical structures of opposites. The goal of this publication is to use the property of contraposition to show that from selected valid forms of logical syllogisms related to graded Peterson's square, we are able to prove valid logical syllogisms related to graded Peterson's cube of opposition. By this procedure we are able to search for valid syllogisms systematically. We also use property of monotonicity of quantifiers to prove other valid syllogisms and order obtained syllogisms. Last but not least we discuss the relationship between valid logical syllogisms obtained by contraposition.

Keywords: Graded Peterson's cube of opposition; Contraposition; Generalized quantifiers; Logical syllogisms with intermediate quantifiers.

Verification of Validity of Logical Syllogisms Generated by Cube of Opposition Using Extended Peterson's Rules

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Special Section: Generalized Quantifiers, Logical Syllogisms and Applications
Track: Papers

Abstract. In this paper, we first advocate for mathematical logic as an indispensable tool for the development of AI. Then we will introduce the concept of intermediate quantifiers and syllogisms with them. Furthermore, we will introduce the graded cube of opposition and suggest extended Peterson's rules for verification of validity of syllogisms with negations. We will prove that all valid syllogisms with negations verify the extended Peterson's rules.

Keywords: Fuzzy natural logic; Intermediate quantifiers; Graded cube of opposition; Extended Peterson's rules; Evaluative linguistic expressions.

Graded Hexagon of Opposition with New Forms of Intermediate Quantifiers

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Special Section: Generalized Quantifiers, Logical Syllogisms and Applications
Track: Abstracts

Abstract. In the presentation, we will form a graded hexagon of opposition as an extension of the graded Peterson's square, incorporating new variations of intermediate quantifiers based on the specific evaluative linguistic expressions. We will start with the classical Aristotle's square of opposition and its extension to hexagon. Then we introduce the graded square of opposition that was suggested by Peterson and fully formalized by the authors. We also introduce quantifiers A few and A little and Several and include them in the latter. Then we introduce new forms of quantifiers and study relations of contrary, sub-contrary, contradictory and sub-(sup-)altern among them. On the basis of the knowledge of the relations among all the intermediate quantifiers we construct the graded Peterson's hexagon of opposition.

Keywords: Square of opposition; Blanché's hexagon of opposition; Fuzzy natural logic (FNL); Graded Peterson's square of opposition in FNL; Intermediate quantifiers; Graded hexagon of opposition in FNL.

Part VIII
Information Fusion Techniques

Insights into the q Exponent in PM with Choquet-Integrals for Classification

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Special Section: Information Fusion Techniques
Track: Papers

Abstract. Choquet-integrals are averaging aggregation functions based on a fuzzy measure that accounts for both the significance of each attribute being aggregated and the interactions between the variables. The effectiveness of a fuzzy measure can be characterized by its accuracy in modeling the relationship or association degree among the elements to be aggregated. In the literature, it is known that amid conventional fuzzy measures, the Power Measure (PM) presents statistically superior performance. This study explores the q exponent impact in the PM on the performance of different Choquet-based integrals when used in fuzzy rulebased classification systems. We aim to analyze how fixed q values influence classification accuracy across thirty-three benchmark datasets. The updated results reveal that smaller q values (e.g., $q = 0.1$ and $q = 0.5$) continue to yield superior accuracy, while larger values ($q \geq 100$) tend to a performance stabilization. Among the tested methods, the generalization named CF1F2-integral achieves the highest classification accuracy, effectively adapting to different parameter settings.

Keywords: Power Measure; Choquet-based integrals; CF1F2-integral; Fuzzy rule-based classification.

Measuring Representativeness through Coverage Degrees and Indexes

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Special Section: Information Fusion Techniques
Track: Papers

Abstract. This paper first introduces the notion of representativeness system (RS) in order to capture the essential aspects of different systems in which a set of sources or representatives are intended to represent or provide a service to a given set of objects. The paper focuses on formally elaborating the notion of coverage indexes as mathematical tools to study and assess the quality of representativeness systems, defining different

subfamilies of this type of indexes as well as providing a construction method for them. The practicality of these indexes is briefly illustrated in the context of cluster quality assessment, highlighting some promising features.

Keywords: Coverage; Representativeness systems; Aggregation functions.

Input Importance in Aggregation Theory by Means of Dependence Stochastic Orders

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Special Section: Information Fusion Techniques
Track: Papers

Abstract. Aggregation functions are usually used to summarize the information from different inputs into a unique value. Depending on the structure of the aggregation function and the behavior of the initial data, there are inputs that have a bigger impact in the result of the aggregation. From a probabilistic approach, such an importance can be identified as the positive dependence between the input and the output. In this paper, sufficient conditions for the stochastic ordering with respect to positive dependence stochastic orders between bivariate random vectors consisting of an input and the output of aggregation functions are provided. In particular, quasi-arithmetic means and some OWA operators are considered, using as ordering the supermodular and concordance stochastic orders.

Keywords: Aggregation; Stochastic orders; Copulas; Dependence.

Understanding Data Properties in the Mallows Model: Impact of Voter Count Variability

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Track: Papers

Abstract. The Mallows model is a sampling technique frequently used to create synthetic election datasets and evaluate ranking aggregation algorithms. It provides flexibility, as authors can recreate different election scenarios by modifying the model's parameters. When applying this model, results from data with different numbers of voters are often compared without adequate consideration, mistakenly assuming a consistent underlying structure in the preference distribution. However, experimental results have shown that this practice may be problematic because some intrinsic properties of the data fluctuate when modifying this parameter. In this paper, we address this issue by studying the behaviour of Mallows model for producing elections when increasing the number of voters. We generate a synthetic dataset that contains simulations of elections for different numbers of voters and alternatives and measure some structural characteristics of the generated data. Afterwards, we analyse the behaviour of these properties as the number of voters increases. Results show that some aspects, such as the modal ranking, are affected by this variation. This leads to the conclusion that the performance of ranking aggregation algorithms, particularly those based on the Condorcet method, cannot be directly compared to data with different numbers of voters due to varying aggregation difficulty.

Keywords: Ranking aggregation; Mallows model; Algorithm evaluation; Condorcet method.

Idempotence and Internality of Aggregations of Random Variables

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Abstract. A classical result states that internality and idempotence are equivalent for aggregation functions. In the context of data analysis, it is natural to consider random variables as the inputs of the aggregation. In this direction, aggregations of random variables are functions that, given a random vector, return a random variable fulfilling monotonicity and some boundary conditions with respect to a stochastic order. This paper is focused on the definition of different notions of idempotence and internality for aggregations of random variables. The implications between the introduced concepts are studied in detail. In addition, families of aggregations of random variables that fulfill the defined properties are provided.

Keywords: Aggregation; Probability theory; Idempotence; Internality.

$[a,b]-FG$-Functionals: A Generalization of Sugeno Integral with Floating Domains in Arbitrary Closed Real Intervals

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Special Section: Information Fusion Techniques
Track: Abstracts

Abstract. The Sugeno integral [1] is an aggregation function that has been successfully applied in different research fields, such as image processing [2]. It has been widely studied and generalized in different forms, such as the notion of Sugeno-like FG-functional (FG-functional) that was proposed by Bardozzo et al. in [2], by replacing the maximum and the minimum of the standard Sugeno integral by other functions (F and G).

However, there are some drawbacks in the definition of an FG-functional. Although it allows positive reals as inputs, the corresponding fuzzy measure is defined on the unit interval. This may cause a significant difference between the magnitudes of the inputs and related fuzzy measure values, so, for some choices of F and G, the inputs may ignored in the calculus of the integral. In fact, Grabisch et al. [3] already pointed out that, regarding the original Sugeno integral, "input vectors and capacities should have the same range", to avoid the output to be cut by the value of the capacity. In FG-functionals, this kind of restriction on the output may occur depending on the choices of functions F and G (e.g., $F \leq \min$ and $G \leq \max$).

The objective of this paper is first to introduce two methods for generalizing the Sugeno integral, both inspired in FG-functionals, allowing inputs in any arbitrary closed real intervals, but overcoming the drawbacks discussed above. For that, we use the constructive approach introduced by Asmus et al. in [4] for defining fusion functions with floating domains in arbitrary closed real intervals $[a,b]$. In the first method, we adjust the magnitudes of the inputs to the range of the considered capacity μ by means of an increasing bijection γ_μ (called here a γ_μ-direct normalization process), and the final aggregated value by an inner function $\mathcal{F}$ recovers its magnitude in $[a,b]$ by the inverse of γ_μ (called here a γ_μ-inverse normalization process). In the second method, the capacity values are adjusted to the domain of the inputs by means of an increasing bijection ζ_μ (ζ_μ-direct normalization process), and there is no need of an inverse normalization process.

We also present different examples and studied several properties of the proposed generalizations, such as monotonicity, idempotency and generalized homogeneity. Then, we show two applications that are made possible by the two proposed generalizations. The first method is used in an edge detection problem, where the inputs are normalized to the range of the considered capacities. The second method is applied in a classification problem, where the capacity is normalized to the range of the inputs.

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Keywords: Sugeno integral; Sugeno-like FG-functionals; ab-fusion functions;

Construction of Uninorms on Bounded Lattices: Augmenting the Flexibility of Choice

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Special Section: Information Fusion Techniques
Track: Abstracts

Abstract. In the field of fuzzy logic and multicriteria decision-making, the construction of aggregation functions such as t-norms, t-conorms and uninorms on bounded lattices has attracted the attention of numerous researchers. Our work focuses on developing methods for constructing uninorms that not only generalize some existing ones, but also provide greater flexibility in the choice of the aggregation functions used within the construction method.

We start by analyzing existing construction methods for uninorms on bounded lattices, identifying patterns and limitations based on the structural properties of the lattices. We observe that traditional construction methods focus on defining t-norms or t-conorms on A set of elements comparable with respect to the neutral element, while on the set of elements incomparable with respect to the neutral element, only the infimum/supremum or simple assessments to the bottom/top element are considered. Although these functions are valid, they considerably restrict the flexibility in information aggregation within the set of incomparable elements with respect to the neutral element. To address this limitation, we propose construction methods for uninorms that not only consider a t-norm and a t-conorm on the intervals for elements dominated and that dominate the neutral element, but also consider a different aggregation function on the set of incomparable elements with respect to the neutral element. Additionally, we explore the possibility of considering uninorms within the subset of elements incomparable with respect to the neutral element. Through a detailed study of the properties of the lattice, we also identify a particular family of lattices that allows us to recursively construct uninorms on blocks of lattices. More precisely, in each block, a uninorm is defined based on a block-wise construction method for uninorms.

These contributions offer more versatile tools for modeling and solving complex problems in fuzzy logic systems and multicriteria decision-making, expanding the list of potential application of aggregation functions on bounded lattices.

Keywords: Uninorms; Bounded lattices; Neutral element; T-norm; T-conorm.

An Analysis of Discrete Fuzzy Negations on a Subset of Discrete Fuzzy Numbers and Their Applications to Discrete T-Subnorms

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Special Section: Information Fusion Techniques
Track: Abstracts

Abstract. Discrete fuzzy numbers on finite chains provide a flexible and computationally efficient framework for modeling uncertainty in systems with limited or ordinal data. Unlike traditional fuzzy numbers, which are defined over continuous domains, discrete fuzzy numbers allow for precise representation and manipulation of uncertainty in applications where data is inherently finite or categorized. They are particularly useful in decision-making, expert systems, and computational intelligence, where linguistic variables and ranked preferences play a crucial role. Applications include medical diagnosis, risk assessment, multi-criteria decision analysis, and pattern recognition, where handling imprecise or vague information is essential for robust modeling and inference.

Recently [1], a total order has been established on the set of discrete fuzzy numbers based on their α-cut representations, providing a structured way to compare and rank them. Additionally, another study [2] has considered the subset of discrete fuzzy numbers defined on a finite chain $L_n = \{0, 1, \ldots, n\}$ with membership values restricted to a finite set $Y_m = \{y_1 = 0 < y_2 < \cdots < y_m = 1\}$, offering a more constrained yet practical approach for applications requiring discrete-valued uncertainty modeling.

If we denote by $\mathcal{A}_1^{L_n \times Y_m}$ the set of discrete fuzzy numbers on the finite chain L_k, where $k+1$ is the cardinality of the set $\mathcal{A}_1^{L_n \times Y_m}$, with membership valuen in Y_m, it is possible to establish a bijection between the set of aggregation functions over $\mathcal{A}_1^{L_n \times Y_m}$ and the set of aggregation functions over the finite chain L_k. Likewise, such a correspondence exists between the set of t-norms and t-conorms defined over $\mathcal{A}_1^{L_n \times Y_m}$ and those defined on L_k, reinforcing the structural connection between discrete fuzzy numbers and classical aggregation and logic operators in finite settings.

In this work, following this line of reasoning, we establish a bijection between the set of negations on $\mathcal{A}_1^{L_n \times Y_m}$ and the set of negations on L_k, allowing us to study the properties of negations on $\mathcal{A}_1^{L_n \times Y_m}$ based on those of negations on L_k. Specifically, we show that there exists a unique strong negation on $\mathcal{A}_1^{L_n \times Y_m}$, introduce the concepts of weak negation and symmetric negation, and establish the equivalence between these two types of negations in $\mathcal{A}_1^{L_n \times Y_m}$ based on their equivalence in L_k.

This theoretical framework allows for a comprehensive study of natural negations associated with discrete t-subnorms in the set $\mathcal{A}_1^{L_n \times Y_m}$, extending the analysis previously conducted for discrete t-subnorms on finite chains [3].

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Keywords: Uncertainty; Discrete fuzzy numbers; Fuzzy Nagations; Aggregation Functions; Discrete t-Subnorms.

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Rule Base Combination in Federated Fuzzy Rule Learning

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Special Section: Information Fusion Techniques
Track: Abstracts

Abstract. Federated Learning aims at building machine learning models in such a way that the data of each of the participating agents is not shared and remains private [1]. In these systems, local learning is performed on each of the agents and then only the parameters of the model are shared and not the original data. Another issue to be addressed in machine learning systems is the ability to explain their results to users. One of the options is the use of transparent models such as fuzzy rule-based systems [2]. In the case of a federated model based on fuzzy rules, the parameters of the model are the rules generated in each client. Unlike other models, such as neural networks, where parameter matching is exact, this is not the case with fuzzy rules. In this case it is necessary to consider the totality of the rule bases generated by each client. In this work we propose a federated fuzzy rule-based learning system that allows the elaboration of explainable systems [3]. In our proposal, the combination of the fuzzy rule bases generated at each client is based on the similarity of the rules.

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Keywords: Federated Learning; Information Fusion; Fuzzy Logic.

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Modelling Sequential Dependent Data with Non-Commutative Aggregation Functions Obtained by Means of Pseudo-Automorphisms

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Special Section: Information Fusion Techniques
Track: Abstracts

Abstract. One property that a significant number of aggregation functions satisfy is commutativity. This property, also called anonimity or symmetry, essentially means that the aggregated value remains invariant regardless of the order in which its arguments are presented. This is required in scenarios where equally significant criteria from anonymous expert opinions are fused.

However, there are cases where commutativity is not only unnecessary but also undesired. This phenomenon can be observed in the processing of sequential-dependent information. An example of this can be observed in neural networks, particularly in the context of processing such data. In this context, the features extracted by neural networks from text, time series, processes or sequences, exhibit a sequential dependence. Consequently, this characteristic of the features or values to be aggregated must be taken into account. In such cases, the order in which values are aggregated is relevant, and also the relative importance of the data is not uniform. Consequently, the use of commutative aggregation functions in this context has an influence that is counter-intuitive and restrictive. The sequential information fusion without the elimination of the axiom of commutativity may result in the disruption of the relationship between the fused values or features.

In this sense, in order to develop methods of information fusion that are not necessarily commutative, some aggregation functions, originally commutative, have been defined in the absence of their commutativity axiom. Examples of such functions include pseudo-t-norms [1], pseudo-t-conorms, pseudo-uninorms [2], pseudo-overlap and pseudo-grouping functions [3].

In this study, non-commutative aggregation functions are constructed by means of pseudo-automorphisms [4], parametrizing them, thus rendering them more adaptable to their use in feature fusion by learning these parameters with an optimization method, such as stochastic gradient descent. This family of functions are then applied to aggregate sequential-dependent features of different neural network architectures in different natural language classification problems. The efficacy of these functions is evidenced by statistically significant enhancements in performance, when contrasted with commutative aggregation functions.

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Keywords: Aggregation functions; Sequential data; Non-commutative aggregation functions; Classification.

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Comparative Study of Choquet-Inspired Aggregation Functions for Dimensionality Reduction in Noisy Images

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Special Section: Information Fusion Techniques
Track: Abstracts

Abstract. Dimensionality reduction is a key task in image processing, particularly relevant in scenarios where storage and computational efficiency are critical, such as in real-time vision systems or embedded devices. Common examples include the pooling layers of convolutional neural networks (CNNs), which reduce spatial dimensions while preserving essential features. In this contribution, we perform a comprehensive comparative study of various aggregation-based operators applied to the dimensionality reduction of grayscale images, with a particular focus on robustness under noisy conditions. Our methodology is based on a sliding window approach, where each 3×3 pixel block is reduced to a single representative value using different aggregation functions [4]. The evaluated methods include classical statistical operators (arithmetic mean, median, minimum, and maximum), the Choquet integral with a fuzzy measure [2] adapted to image data, and a family of d-Choquet Inspired operators that combine dissimilarity functions and aggregation strategies in a flexible and expressive manner [3]. We also consider an approach based on moderate deviation principles [1]. Furthermore, we introduce an adaptive framework that locally selects the best d-Choquet Inspired configuration based on a penalty function, allowing each window to be processed with the operator that minimizes the local reconstruction error. To evaluate performance, we employ multiple quality metrics: Structural Similarity Index (SSIM) [5], a custom similarity measure (SM), mean squared error (MSE), and a hybrid penalty-based error function. All experiments are conducted using a set of natural images, both in their original and noise-corrupted versions (Gaussian, salt-and-pepper, Poisson, and speckle noise). The reduced images are linearly interpolated to their original size to facilitate direct comparison with the ground truth.

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Keywords: Dimensionality reduction; Image processing; Aggregation functions; Choquet integral; d-Choquet Inspired Operators.

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Global Pooling of Features in Convolutional Neural Networks through Moderate Deviation Based Aggregation Functions

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Special Section: Information Fusion Techniques
Track: Abstracts

Abstract. Obtaining the "best representative" for a set of input values is one of the most recurrent problems in information fusion. Functions such as penalty-based functions [1, 2] or Daróczy means [3] try to solve this problem, although they face several limitations. Daróczy means, in particular, do not always remain monotonic, and therefore are not aggregation functions in general. A similar approach to the construction of Daróczy means can be followed using the concept of moderate deviation functions [4], which ensures that the resulting function is an aggregation function. Several constructions for aggregations based on moderate deviation functions have been presented through the years [5, 6]. In particular, in [7], the problem of aggregating values in the range $[-1, 1]$ was tackled. Here, we present an extension of such approach, which allows to aggregate values in a general range $[a, b] \subset \mathbb{R} \cup \{-\infty, \infty\}$ by isomorphism from the interval $[a, b]$ to $[-1, 1]$. We also show how such functions can be useful as fusion operators for machine learning tasks which work with real valued data, such as neural networks. In particular, we show that aggregations constructed through moderate deviation functions can improve upon the classical Global Average Pooling operator [8] used by Convolutional Neural Networks to fuse the features extracted by convolutional layers, without causing any noticeable overhead.

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Keywords: Moderate deviation function; Aggregation function; Convolutional neural network; Global pooling.

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Uncertainty Fusion in Generalizations of Fuzzy Sets Using Distance Transformations

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Special Section: Information Fusion Techniques
Track: Abstracts

Abstract. Fuzzy sets have been widely used to model uncertainty in various contexts, whose specific conditions have fueled the need for the so-called generalizations of fuzzy sets [1]. Generalizations of fuzzy sets are normally developed to accomodate different expressions of uncertainty, evolving from scalar membership degrees (in the original fuzzy sets) to functional or set-valued membership degrees (in type-2 fuzzy sets or set-valued fuzzy sets, respectively). This wide range of representations have subsequently led to a large literature on operators and functions designed to accomodate most of the classical concepts in fuzzy set theory to each individual generalization.

While the richness in adapting notions from fuzzy sets to generalizations of fuzzy sets is remarkable, it is also true that it prevents an homogeneous interpretation of fuzzy set theory. We believe that, instead of studying operators able to deal with the specific conditions of each generalization of fuzzy sets, an effort should be made to develop frameworks that accomodate such generalizations. Such frameworks could be further used to produce unified operators and functions applicable to many, if not all, possible generalizations of fuzzy sets. In this regard, we propose the use of distance transformations [2] to produce functional representations of membership degrees, accomodating (at least) fuzzy sets, interval-valued fuzzy sets and set-valued fuzzy sets. We present the fundamentals of our proposal, as well as some prospective applications in the context of fusion functions.

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Keywords: Fuzzy sets; Generalizations of fuzzy sets; Distance transformations; Uncertainty fusion.
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Capturing Divergent ESG and Financial Priorities in Heterogeneous Investor Groups: An Opinion Dynamics-Based Multi-Criteria Decision-Making Approach

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Special Section: Information Fusion Techniques
Track: Abstracts

Abstract. Heterogeneity in investors' prioritization of Environmental, Social, and Governance (ESG) and financial criteria poses significant challenges for collective decision-making in group investment contexts. This study proposes a novel decision framework that models how social interactions dynamically influence investors' prioritization of decision criteria and their evaluation of investment alternatives. Before initiating the group decision-making process, the weights of ESG sub-criteria and the financial performance indicators of portfolio stocks are computed using a new method called Linguistic Probabilistic Step-Wise Weight Assessing Ratio Analysis (LP-SWARA), which incorporates each investor's prioritization. Subsequently, the weights of the main ESG

and financial criteria are determined through the Möbius representation of a fuzzy measure. Each investor then evaluates each stock subjectively under each criterion and fuses these subjective judgments with objective evaluations obtained via a sigmoidal membership function. After the group decision-making phase begins, a linguistic probabilistic Bayesian network is employed to construct the social trust network, addressing issues of missing data in trust elicitation while enabling investors to express trust levels using a probabilistic linguistic term set naturally. The resulting trust matrix is transformed into a dominance matrix, which forms the foundation of an opinion dynamics model that simulates the evolution of investor preferences through iterative consensus-building steps. At each iteration, a collective preference matrix is generated, and each investor quantifies the divergence between their individual opinion and the group consensus using a symmetric fuzzy cross-entropy measure. These divergences are then aggregated via the Choquet integral, which allows for non-additive synthesis of disagreement so it is a new type of consensus measure which is called the Choquet distance measure based on fuzzy cross entropy Consensus on an investment alternative is declared once the Choquet-integrated fuzzy cross-entropy falls below a predefined threshold; otherwise, the opinion dynamics process continues to promote further alignment. The proposed method provides a linguistically expressive, socially adaptive, and mathematically rigorous mechanism for multi-criteria group investment decision-making, particularly effective in contexts where ESG and financial priorities are divergent across investors.

Keywords: Social Trust Network; Linguistic Probabilistic Term Set; Fuzzy Cross Entropy; Möbius Representation; Financial Portfolio Optimization.

Learnable Fuzzy Measures for Enhancing F-Integrals in Deep Learning

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Special Section: Information Fusion Techniques
Track: Abstracts

Abstract. Discrete Fuzzy Integrals, F-integrals, have been successfully implemented as fusion functions in several neural network architectures, including LSTMs [1] and CNNs [2]. Their ability to capture high-order dependencies makes them especially powerful for interdependent data. However, their practical use has been limited to low-dimensional problems due to scalability issues [3].

Computing an F-integral for n elements involves measuring n coalitions. However, the specific coalitions required depend on the magnitude of the input data and are not known in advance. As a result, the entire fuzzy measure is often precomputed by evaluating all 2^n subsets, which becomes impractical as n grows.

To overcome this limitation, we construct fuzzy measures dynamically during inference, eliminating the need for full precomputation. Building on this approach, we propose a gradient-based framework for integrating learnable fuzzy measures into neural networks, with a focus on Graph Neural Networks (GNNs). Incorporating F-integrals with trainable measures into GNNs broadens the traditional family of aggregation operators, enhancing their capacity to model complex interactions between nodes.

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Keywords: Fuzzy measure; Aggregation function; Graph neural network.

Part IX
Interval Uncertainty

A New Approach to Interval Clustering Using Ordering Degrees

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Special Section: Interval Uncertainty
Track: Abstracts

Abstract. The problem of comparing intervals arises frequently across mathematical and computational domains, yet conventional ordering techniques often fall short due to the inherent incomparability present in partial orders. Building on previous work [1,2,3], we use ordering degrees, real-valued functions designed to quantify how strongly one interval is considered smaller than another within a partial order. We concentrate on two central ordering frameworks: the lattice order and the content inclusion order. We propose a method to construct ordering degrees based on functions that satisfy quasi-distance properties. These ordering degree functions not only capture directional dissimilarity but also preserve essential order-theoretical properties. The construction is inspired by concepts originally developed in the context of interval-valued fuzzy sets, adapted here to work with classical intervals.

The framework supports an epistemic interpretation of intervals, allowing for graded relational reasoning that reflects degrees of knowledge or uncertainty.

We derive similarity measures by symmetrizing the underlying quasi-distances. These similarity measures capture both the directional and structural resemblance between intervals and can be employed in a hierarchical clustering algorithm. Preliminary results on both analytically constructed scenarios and real-world datasets suggest that the framework is interpretable and shows promising behavior compared to existing interval clustering similarities [4].

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Keywords: Ordering degree; Quasi distance; Partial order; Similarity measures; Hierarchical clustering.

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On the Definition of Arithmetic Operations for Intervals under the Xu and Yager Order

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Special Section: Interval Uncertainty
Track: Abstracts

Abstract. Intervals are widely used to represent uncertain or imprecise data, especially in contexts where human judgment or subjective assessments are involved. Working with intervals requires performing arithmetic operations, particularly addition and subtraction. While the sum of two intervals, defined as the sum of their endpoints, is well-defined, a similar approach to subtraction is not always valid. For example, $[3, 4] - [1, 7] = [2, -3]$. This limitation motivates the need for a new arithmetic framework capable of handling such operations in a consistent and interpretable way.

From an epistemic perspective [1], intervals are seen as approximate representations of real values that cannot be precisely determined. In this work, we model intervals as uniformly distributed random variables, under the assumption that all values within the interval are equally likely. Based on this interpretation, we propose new definitions for the addition and subtraction of intervals that preserve key probabilistic properties. Specifically, we define these operations so that the result remains a uniform distribution, preserving both the expected value and variance of the resulting trapezoidal distribution obtained from the classical convolution of two independent uniform distributions [2]. Additionally, our definitions are consistent with the Xu and Yager order for intervals [3]. We also explore several mathematical properties of these new operations, aiming to contribute to the development of more robust tools for reasoning with interval-valued data. Finally, we apply our framework to the resolution of the linear assignment problem where costs are given as intervals, illustrating the practical value of our approach in decision-making under uncertainty.

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Keywords: Interval data; Uniform distribution; Interval arithmetic; Xu and Yaguer order; Linear assignment problem.

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Part X
Mathematical Fuzzy logic

On Some Properties of Tabular Varieties of MTL-Algebras and Their Decidability

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Special Section: Mathematical Fuzzy Logic
Track: Papers

Abstract. A variety is called tabular (T) whenever it is generated by one finite algebra. A pretabular (PT) variety is a non tabular variety whose proper subvarieties are all tabular. Those notions were firstly studied for Heyting algebras, in the sixties and seventies. An important role of PT varieties is to check if the tabularity problem (i.e. deciding if a certain variety is tabular or not) is decidable. In a recent work we started studying T and PT varieties of MTL-algebras, finding some preliminary results. In this paper we take a step further, by studing the decidability issues for some properties for tabular varieties of MTL-algebras, like tabularity, consistent tabularity, non-boolean tabularity, single chain generated and amalgamation property. We study the decidability and the computational complexity of those problems.

Keywords: MTL-algebras; Tabular Varieties; Pretabular Varieties; Decidability; Tabularity problem; Amalgamation Property; Single-chain completeness.

Foulis Quantales and Complete Orthomodular Lattices

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Special Section: Mathematical Fuzzy Logic
Track: Papers

Abstract. Our approach establishes a natural correspondence between complete orthomodular lattices and certain types of quantales.

Firstly, given a complete orthomodular lattice X, we associate with it a Foulis quantale $\mathrm{Lin}(X)$ consisting of its endomorphisms. This allows us to view X as a left module over $\mathrm{Lin}(X)$, thereby introducing a novel fuzzy-theoretic perspective to the study of complete orthomodular lattices.

Conversely, for any Foulis quantale Q, we associate a complete orthomodular lattice $[Q]$ that naturally forms a left Q-module. Furthermore, there exists a canonical homomorphism of Foulis quantales from Q to $\mathrm{Lin}([Q])$.

Keywords: Quantale; Quantale module; Orthomodular lattice; Linear map; Foulis semigroup; Dagger category;

Gödel Logic and Nested Regular Languages

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Special Section: Mathematical Fuzzy Logic
Track: Abstracts

Abstract. In [1] the authors introduce the notion of fortresses as recognisers of regular languages. A fortress mimics the behaviour of a deterministic finite state automaton by means of classical propositional logic concepts. In particular, a fortress is a triple $(\varphi, \{s_a : a \in \Sigma\}, \Theta)$, where Θ is a prime theory, each s_a (for "a" symbol of the alphabet Σ) is a substitution mapping variables to formulas, and φ is a formula over the first n variables. A fortress accepts a word $w = a_1...a_k$ if and only if the formula $s_{a_1}(s_{a_2}(\dots(s_{a_k}(\varphi))\dots))$ is logical consequence of the theory Θ. It is proven that fortresses accept exactly the class of regular languages. If we replace Θ with a general theory Γ, the fortress so defined accepts the intersection of all the languages corresponding to the prime theories containing Γ. In this work we generalise the notion of fortress to Gödel infinitely valued propositional logic simply by interpreting the logical consequence relation in Gödel logic instead of classical Boolean logic. Using a categorical duality, we show that fortresses interpreted in Gödel logic recognise exactly the class of nested regular languages, that is, sequences of regular languages $L_1 \subset L_2 \subset \cdots \subset L_m$. In this light, the use of Gödel logic in recognising formal languages naturally proposes a logically supported notion of fuzzification of regular languages, as each L_i can be considered an alpha-cut of a fuzzy regular language.

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Keywords: Gödel logic; Finite state automata; Gödel algebra.

On the Non-Falsity and Threshold Preserving Variants of MTL Logics

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Special Section: Mathematical Fuzzy Logic
Track: Papers

Abstract. In this paper we study the definition and axiomatisation of non-falsity preserving and threshold preserving companions of several extensions of the Monoidal t-norm based fuzzy logic MTL. More in detail, we first extend some recent preliminary results on non-falsity preserving logics, and then we present a new study on threshold-preserving companions of the main three fuzzy logics, Łukasiewicz, Product and Gödel logics.

Keywords: Mathematical fuzzy logic; MTL; Non-falsity preserving logics; Threshold-preserving logics.

Quantitative Lockean Thesis and Its Logical Representation

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Special Section: Mathematical Fuzzy Logic
Track: Papers

Abstract. The present paper considers a generalization of the Lockean thesis to a quantitative, many-valued setting. The aim of the proposed many-valued generalization is to handle the conjunctive closure principle, that usual fails in the classical setting, by a gradual approach. Being the Lockean thesis probabilistic in nature, we also show how its quantitative version can be formalized within the language of the probability logic FP(RL). Our analysis shows that the belief operator definable in FP(RL) recover the satisfiability of belief sets that might be classically contradictory. In other words, there are belief set whose conjunctive closure is classically unsatisfiable, but whose generalized representation in FP(RL) is satisfiable.

Keywords: Lockean thesis; Probability logic; Conjunctive closure.

A First-Order Extension of Fuzzy Probability Logics

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Special Section: Mathematical Fuzzy Logic
Track: Abstracts

Abstract. Probability logics provide rigorous methods for formalizing uncertain reasoning. Probability logics that exploit the machinery of fuzzy logic, or fuzzy probability logics, were introduced by Esteva, Godo, and Hajek (1995) and further developed by various authors, including Franco Montagna (2005, 2011). In this style of formalizing probabilistic reasoning, the apparatus of a suitable fuzzy logic is applied to $[0,1]$-valued modal atoms $P(A)$, informally interpreted as the graded statement "the event A is probable". Several variants of fuzzy probability logic have been studied, including those suitable for conditional probabilities (using a binary fuzzy modality $P(A|B)$), for more general structures of probability values (e.g., hyperreal), and with different strengths of the underlying fuzzy logic. However, all of the existing fuzzy probability logics have remained just propositional, which significantly limits their ability to represent quantified statements and complex probabilistic concepts.

This presentation introduces a predicate variant of two-layered fuzzy probability logic, extending the propositional fuzzy probability logic FP(L) to the first-order setting FP(L1), where L is Lukasiewicz fuzzy logic (or a well-behaved expansion thereof) and L1 is its first-order variant. This predicate extension employs a two-layered syntax, distinguishing between formulas for crisp events and probabilistic formulas evaluated using the truth values in the real unit interval provided by the background fuzzy logic. The proposed framework also integrates crisp non-nestable S5-style alethic modalities (of necessity, possibility, and actuality) to enhance expressive power.

The $[0,1]$-valued (or "standard") semantics of FP(L1) is based on finitely additive probability measures over collections of first-order models (or "possible worlds") sharing a common domain of individuals. Axiomatic completeness for FP(L1) can be established via translation to the first-order fuzzy logic L1, following a known method from the propositional case. Among the valid logical laws, FP(L1) ensures the provability of classical properties of finitely additive probability measures, incorporates the laws of crisp S5-style alethic modalities, and exhibits distinct properties concerning quantification and probabilistic operators: for example, the converse Barcan formulas are valid for the modality P, while the Barcan formulas themselves are not.

Finally, potential extensions of the first-order fuzzy probability logic FP(L1) will briefly be discussed, including generalizations to handle conditional probabilities, non-constant domains of individuals, and multi-agent

settings, hinting at the framework's versatility in formalizing probabilistic reasoning as well as its applicability in game-theoretic, doxastic, and deontic contexts.

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Keywords: Fuzzy logic; Probability logic; Predicate logic; Lukasiewicz logic; Modal logic.

Part XI
Modeling Complex Dynamics: Adapting Analytical Tools for Diverse Scenarios

Adaptive Fuzzy Level Set Algorithm for Bitcoin Realized Volatility Modeling and Forecasting

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Special Section: Modeling Complex Dynamics: Adapting Analytical Tools for Diverse Scenarios
Track: Papers

Abstract. This paper addresses a novel approach to cryptocurrency risk management. An adaptive fuzzy model based on level sets is suggested to model and forecast the realized volatility of Bitcoin. The model, referred to as adaptive level set model (ALSM), is a rule-based fuzzy inference system that uses the concept of level sets to determine the model output in a data-driven and adaptive manner. One-step-ahead forecasts of realized volatility generated by the ALSM model are evaluated in terms of accuracy and compared to alternative machine learning models, an evolving fuzzy model, and the heterogeneous autoregressive (HAR) model. HAR serves as the baseline for realized volatility forecasting evaluation. The results indicate that the ALSM model achieves the highest accuracy among all competing approaches, highlighting its potential as a valuable tool to assist investors in forecasting risk in Bitcoin market.

Keywords: Adaptive fuzzy modeling; Realized volatility; Level set; Bitcoin; Forecasting.

Unconstrained Parametrization of Proper Symplectic Decomposition for Learning Hamiltonian Dynamics

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Special Section: Modeling Complex Dynamics: Adapting Analytical Tools for Diverse Scenarios
Track: Papers

Abstract. Symplecticity-preserving neural networks such as SympNets have been proposed to learn the flow of symplectic Hamiltonian dynamics and to obtain qualitatively better long-time predictions. Computationally, learning high-dimensional problems still poses a great challenge. Structure-preserving dimensionality reduction methods have been developed to improve computational efficiency, such as the proper symplectic decomposition (PSD), to preserve the inherent geometric property of the system when learning Hamiltonian dynamics. Several near-optimal PSD solutions, such as a cotangent lift solution, have also been constructed. In this work, we propose a symplecticity-preserving unconstrained parametrization of the symplectic lift matrices, such that the dimensionality reduction can be learned simultaneously with learning Hamiltonian dynamics in the dimension-reduced phase space. With this approach, we obtain more accurate numerical results, especially long- time predictions, compared to learning dimension-reduced dynamics with the previously introduced constant PSD cotangent lift solution.

Keywords: Hamiltonian systems; Structure-preserving neural networks; Proper symplectic decomposition; Dimensionality reduction.

Multiplication and Linear Interactivity

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Special Section: Modeling Complex Dynamics: Adapting Analytical Tools for Diverse Scenarios
Track: Abstracts

Abstract. This work is focused on arithmetic with fuzzy numbers, specifically, on the operation multiplication as an inversion operation for division. The inversion operation is essential in interval and fuzzy arithmetic and analysis, for example, in the concepts of differentiability, in applications to the solution of integral and differential equations, etc. Multiplication governed by Zadeh's extension principle, as all traditional fuzzy arithmetic operations, assumes independence between operands. However, in many practical situations, fuzzy numbers are not independent but interact, leading to the notion of interactive fuzzy numbers and interactive fuzzy arithmetic. We define and analyze interactive multiplication obtained from a sup-J extension principle which is a generalization of Zadeh's extension principle, where J is a fuzzy relation between fuzzy numbers representing the interactivity. In particular, we work with the fuzzy relation called complete correlation, which represents linear interactivity, and we follow up our previous research about division operation for completely correlated fuzzy numbers. We investigate properties of this interactive multiplication and describe conditions for invertibility between interactive multiplication and division for completely correlated fuzzy numbers. In addition, we explore various multiplication operations and provide a theoretical and practical comparison with the proposed interactive multiplication.

Keywords: Interactive fuzzy numbers; Complete correlation; Interactive multiplication; Interactive division.

Differential Equations Driven by Fuzzy Measures – Existence and Uniqueness of Solution of Initial Value Problem

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Special Section: Modeling Complex Dynamics: Adapting Analytical Tools for Diverse Scenarios
Track: Abstracts

Abstract. In this post, we will investigate the existence of a solution to the initial value problem of the first-order differential equations with respect to non-additive mea- sures (fuzzy measures) by generalization from the case of distorted Lebesgue mea- sure, from [1]. Where the derivative of the unknown variable is the Choquet-Radon-Nikodym derivative w.r.t. fuzzy measure, described in more detail in [2].

Then, with some assumptions, we can express the differential equation w.r.t. non- additive measures in terms of ordinary derivatives, and the resulting expression could be understood as an operator. Afterward, we will investigate this operator by using Banach fixed point theorem and Picard's-Lindel¨of theorem, as well as some well-known inequalities from functional analysis (Holder's inequality, Gener- alized Minkowski inequality, Cauchy-Schwarz inequality), we prove the existence and uniqueness of nonnegative solutions, under some reasonable conditions. In the end, we can discuss the possibility of choosing other types of Choquet integrals, further described in [3], to get different operators that should have similar properties.

Unfortunately, we do not know if this approach could describe the solutions of linear differential equations w.r.t. non-additive measures (without some assumptions on the solution, which is very limiting). The solutions of such equations are well-known (see [4,5]), but we are not able to determine whether they are unique or not.

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Keywords: Fuzzy measures; Measure-driven differential equations; Choquet integral; Choquet-Radon-Nikodym derivative; Initial value problem.

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Lower–Upper Bounds Approach to Solve Differential Equations: Theory, Universal Properties, and Numerical Methods

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Special Section: Modeling Complex Dynamics: Adapting Analytical Tools for Diverse Scenarios
Track: Abstracts

Abstract. We develop a novel fuzzy set-based framework for approximating solutions to differential equations by defining lower and upper bounds for real-valued functions. We extend the lattice-based F-transform [1] (adapted in [2]), and provide a solid mathematical basis. We demonstrate universal approximation properties, showing that any continuous function can be enclosed within dedicated bounds with arbitrary precision. Numerical methods are introduced for stable and convergent solutions, including adaptations to solve Initial Value Problems (IVP) and Fuzzy Initial Value Problems under Hukuhara and generalized Hukuhara differentiability.
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Keywords: Fuzzy sets; Lower approximation; Upper approximation; Fuzzy differential equations.

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Part XII
New Contexts in Aggregation Theory

Ontology Aggregation with Maximum Consensus Based on a Fuzzy Multi-Criteria Group Decision Making Method

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Special Section: New Contexts in Aggregation Theory
Track: Papers

Abstract. In this work, we extend the integration of fuzzy logic in Multi-Criteria Decision-Making (MCDM) problems and its application to ontologies. We define an MCGDM framework where experts assign scores and weights to ontology classes, and each one of them is assigned a fuzzy weight, reflecting their relative importance in the decision process. Each expert select their best choice among the alternatives and a final best compromise A^* is derived using a minimal mean distance operator, ensuring that the aggregated result optimally reflects expert opinions while minimizing deviations from individual preferences.

Keywords: Multi-Criteria Group Decision-Making; Fuzzy Ontology; Fuzzy method; Fuzzy multi-expert decision-making.

An Axiomatic Study of the Properties Satisfied by Methods for Ranking the Elements of a Poset

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Special Section: New Contexts in Aggregation Theory
Track: Papers

Abstract. In discrete mathematics, an interesting problem that has called the attention of many scholars is that of ranking the elements of a given partially ordered set. In this contribution, we propose reasonable properties that a method for ranking the elements of a poset may satisfy and we study the relationships between these properties. Interestingly, it is shown how a plethora of additional properties may be borrowed from the field of social choice theory and, in particular, from the problem of the aggregation of rankings. Finally, we present three prominent methods for ranking the elements of a poset (namely, the averaged rank method, the mutual rank probabilities and the maximal method) and analyse which among the proposed properties each of these three methods satisfies.

Keywords: Poset; Averaged rank method; Mutual rank probabilities; Maximal method; Aggregation of rankings.

Goodness-of-Fit Tests to Location-Scale Families Based on OWA Functions

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Special Section: New Contexts in Aggregation Theory
Track: Papers

Abstract. Skewness coefficients are measures for quantifying the degree of asymmetry of a random variable. Different authors have proposed several skewness coefficients, most of which are positioned within the axiomatic definition introduced by Oja. This contribution presents a general family of skewness coefficients based on OWA functions. After showing that this family fits within Oja's axiomatic definition of a skewness coefficient, we present a sample version of this coefficient, study its asymptotic distribution, and use it for defining a goodness-of-fit test to a location-scale family.

Keywords: Skewness coefficient; OWA function; Goodness-of-fit; Location-scale family.

Multidimensional Aggregation and Social Poverty

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Special Section: New Contexts in Aggregation Theory
Track: Papers

Abstract. Multidimensional poverty and deprivation measures are increasingly used in academic research and policy evaluations. Recognizing the multidimensional nature of well-being and poverty is essential for these assessments. This paper explores various categories of social poverty and deprivation measures, employing an axiomatic framework to analyze cases where the only information available for each attribute is whether an individual is deprived of it or not.

Keywords: Multidimensional poverty; Multidimensional inequality; Supermodular functions; OWA; OWMax; SMSD indicator.

Decomposition of Non-Commutative Associative Functions via Non-Commutative Ordinal Sum

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Special Section: New Contexts in Aggregation Theory
Track: Abstracts

Abstract. In [2], we have shown that all commutative associative aggregation functions, which are continuous around the main diagonal can be decomposed into a z-ordinal sum of representable and idempotent semigroups, and since each commutative idempotent semigroup can be decomposed into z-ordinal sum of trivial semigroups, we obtain the following result. Each commutative associative aggregation function, which is continuous around the main diagonal can be expressed as a z-ordinal sum of a countable number of semigroups corresponding to t-norms, t-conorms and uninorms with continuous additive generators, and a possibly uncountable number of

trivial semigroups. This result covers the characterizations of all basic functions such as continuous t-norms, t-conorms, uninorms with continuous underlying functions and n-uninorms with continuous underlying functions. However, recent developments in the field of aggregation theory have experienced an increased demand for non-commutative functions required in applications. The reasons for this demand are various: in some applications the commutativity is superfluous, or even too restrictive, while in others a commutative aggregation function simply cannot fully capture a non-commutative nature of the studied problem. As a response to this problem, several characterizations of basic non-commutative associative aggregation functions, such as pseudo-uninorms with continuous underlying functions, or pseudo-n-uninorms with continuous underlying functions have been shown (see, e.g., [1,4]). Nevertheless, in the characterization of more general non-commutative associative aggregation functions it is obvious that standard decomposition methods, such as ordinal and z-ordinal sum, that can be used for decomposition of their commutative counterparts are no longer suitable for decomposition of non-commutative functions. Therefore, we have introduced a non-commutative ordinal sum construction in [3], which generalizes (z-)ordinal sum to the non-commutative case. In this contribution, we would like to discuss the structure of several distinguished non-commutative associative aggregation functions defined on the unit interval. We will show their decomposition via non-commutative ordinal sum and the pair-order induced by the related associated idempotent function. Our goal is to achieve a similar decomposition as in the case of commutative aggregation functions, i.e., such in which summands are only representable and trivial semigroups. We will also highlight the differences and similarities between the commutative and the non-commutative case and show which non-commutative functions can be decomposed also via standard ordinal sum.

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Keywords: Semigroup; Ordinal sum; Order; Pseudo-n-uninorm.

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Exploring Divisibility and Related Properties of T-Norms on Finite Lattices

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Special Section: New Contexts in Aggregation Theory
Track: Abstracts

Abstract. In [1] authors discussed t-norms on discrete chains, particularly on finite chains. Since on the finite setting, the continuity as the crucial topological property is absent, they focused on three properties, which could be considered as an extension of continuity in this setting. Moreover, for t-norms on finite chains these properties are equivalent. In our contribution, we would like to extend these notions to finite lattices and point out some notable differences, which occur here.

Note that infimum on each lattice is divisible, but there are finite lattices, where infimum is not smooth and even finite lattices, where infimum does not satisfy the intermediate value property, which means that from this point of view general finite lattices significantly differ from finite chains.

The divisibility of a defined function on a finite chain together with the monotonicity, associativity and the neutral element 1 immediately imply commutativity [2]. However, we propose a new construction method leading to a new class of divisible proper pseudo-t-norms, which are non-commutative counterparts of t-norms.

Finally, similarly to continuous t-norms on the unit interval, the class of divisible t-norms has been characterized as an ordinal sum of divisible (continuous) t-norms (see [1, 3]). Since then several other construction methods, extending ordinal sum were proposed [4, 5], we discuss which of them is suitable for construction of

divisible t-norms.

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Keywords: Divisible t-norm; Smooth t-norm; Intermediate value-property; Finite lattice; Infimum;

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Decomposition Integrals Constrained by Linear Inequalities

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Special Section: New Contexts in Aggregation Theory
Track: Abstracts

Abstract. In this contribution, we introduce a novel modification of decomposition integrals, a class of integrals originally proposed by Even and Lehrer [1]. Decomposition integrals provide a unifying framework that encompasses several well-known integrals, including the Choquet integral, the PAN integral, the Shilkret integral, and concave integral, making them a subject of significant interest. Various modifications of decomposition integrals have been studied, such as those for interval-valued functions [2], greedy decomposition integrals [3], and minimax decomposition integrals [4].

Our proposed modification further refines the decomposition process by imposing additional constraints in the form of linear inequalities, in addition to the underlying decomposition system. These inequalities enable the modeling of more realistic scenarios by allowing for the restriction of available resources or the enforcement of minimal combinations of certain components, among other applications.

We explore the fundamental properties of these modified decomposition integrals and provide illustrative examples to demonstrate their applicability.

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Keywords: Decomposition integral; Aggregation function; Linear inequalities.

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A New Perspective on the Discrete Choquet Integral and OWA Operator

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Special Section: New Contexts in Aggregation Theory
Track: Abstracts

Abstract. The Ordered Weighted Averaging (OWA) operator, introduced by Yager [4], occupies a central place in aggregation theory due to its capacity to model a wide range of attitudes—from optimism to pessimism. Formally, the OWA operator is defined as a weighted sum of ordered inputs, placing it within the class of symmetric and averaging aggregation functions. Its flexibility in encoding various decision-making strategies through the adjustment of weight vectors has made it particularly influential across fields such as decision theory, fuzzy logic, and artificial intelligence. Nevertheless, certain extensions and generalizations of this operator remain underexplored.

In this contribution, we build upon a lesser-known formula that serves as a foundation for further extensions of the discrete Choquet integral via conditional aggregation operators [2]. This representation first appeared in [1, formula (4.5)] and was later presented in [3], where it was proven using Schmeidler's characterization of the Choquet integral. Since it is well established that the Choquet integral with respect to a symmetric capacity coincides with the OWA operator, this alternative representation naturally leads to a novel expression of the OWA operator itself. We investigate the fundamental properties of this generalized form and compare them with those of the classical OWA operator. This analysis not only sheds light on the structural flexibility of OWA-type operators but also opens pathways for new applications in fuzzy systems and aggregation-based modeling.

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Keywords: Discrete Choquet integral; OWA operator; Aggregation.

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A General Framework for Measuring Chord Distances Using Aggregation Functions

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Special Section: New Contexts in Aggregation Theory
Track: Abstracts

Abstract. Measuring distances between musical chords presents a complex challenge, particularly when chords differ in size and internal structure. Traditional methods—such as the Hausdorff distance [5], Fujita's method [2], the Jaccard index [3], and the symmetric difference counting measure [1]—offer valuable perspectives but also exhibit inherent limitations. To overcome these challenges, we introduce a general framework based on aggregation functions, capable of expressing a broad class of chord distance measures within a unified formalism.

This endeavor stems from the inherent complexity of musical structures. Chords, as foundational elements of harmonic language, are naturally modeled as sets of musical tones, which may vary both in cardinality and pitch content. Consequently, the notion of distance between chords is intimately connected to how we conceptualize the relationships among their constituent tones. For instance, the Hausdorff distance reflects the "maximum deviation" between two sets, while Fujita's method focuses on an "average" inter-set distance. Although each approach sheds light on certain aspects of musical similarity, none provides a universally adequate solution—particularly in the context of complex, real-world chord progressions.

To address this gap, we propose a more adaptable and expressive approach using aggregation functions. These functions enable the combination of multiple distance measures into a single, unified (semi-)metric, thereby accommodating various musical features simultaneously. This aggregation-based methodology allows for the weighting of specific characteristics according to their relevance, which is essential when analyzing chords of differing complexity. As a result, our framework not only synthesizes existing distance measures but also enhances our capacity to model harmonic similarity with mathematical rigor and musical nuance.

Our aim is not merely to bridge mathematical theory and musical structure, but more specifically to demonstrate how soft aggregation techniques and operator-based modeling can serve as powerful tools for interpreting symbolic musical information—particularly within the framework of fuzzy logic. This perspective opens new pathways for nuanced, flexible analysis of harmonic content, supporting both theoretical insights and practical applications in music information retrieval, analysis, and composition.

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Keywords: Distance; Metric space; Aggregation functions; Chord.

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Part XIII
Representing and Managing Uncertainty

On Direct Systems of Implications with Graded Attributes

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Special Section: Representing and Managing Uncertainty
Track: Papers

Abstract. In this paper the problem of defining direct systems of implications in the fuzzy setting is studied. The directness of systems allows a quick computation of the closure operator in cases such as Fuzzy Formal Concept Analysis. Characterizing these properties in algebraic terms is deeply linked to Simplification Logic. After the theoretical results, some thoughts on algorithms to provide direct systems are also considered.

Keywords: Fuzzy Formal Concept Analysis; Bases of implications; Simplification Logic.
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Studying the Structure Generated by Subsets of φ-Indexes of Inclusion

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Special Section: Representing and Managing Uncertainty
Track: Abstracts

Abstract. Research on fuzzy inclusions is of significant interest due to its broad applications in fields such as decision-making, pattern recognition, and knowledge representation. Different approaches to the notion of fuzzy inclusions can be found in the literature, but all of them have a common feature: they provide a numerical value to represent the degree of inclusion of one fuzzy set into another. The (functional) φ-index of inclusion was presented originally [1] as a novel approach to model the inclusion between fuzzy sets and it was extended to general L-fuzzy sets [2]. The main difference with respect to the existing approaches in the literature is that the inclusion between fuzzy sets are represented by mappings, instead of by values in the lattice of truth degrees.

We will focus on the properties of the φ-index of inclusion, when its definition is restricted to a subset of indexes. The theoretical results obtained in this work are necessary in order to develop fuzzy inference systems based on it for the different uses of fuzzy reasoning in real-world applications.

The results obtained so far for the φ-index of inclusion motivate its use for developing fuzzy inference systems. With this goal in mind, we have noticed that it is necessary to analyze the properties of the φ-index of inclusion when its definition is restricted to a subset of indexes. This talk provides the first steps in this direction.

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Keywords: Fuzzy inclusion; Index of inclusion; Fuzzy inference systems.

Idea Management and Game Theory with Uncertain Payoffs

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Special Section: Representing and Managing Uncertainty
Track: Papers

Abstract. In conflict situations, which are considered in game theory, the decision is made not by one individual, but by several participants, and the payoff function for each individual depends not only on his strategy, but also on the strategies of the other participants. Idea management is a system that helps to generate, develop, and implement ideas. Organizations usually use idea management systems to supplement their innovation management processes. Idea management takes place at the forefront of innovation management. One of the stages of the idea management process is the acceptance and rejection of ideas. Idea generators and idea acceptors can be seen as two players with possibly different opinions. The payoffs of these players are of an uncertain nature. This article will offer a solution on how to make a decision that could satisfy both players.

Keywords: Idea management; Normal-form game; Extensive-form game; Nash equilibrium; Uncertain payoff.

Computing Minimal Solutions of Fuzzy Relation Equations with Incremental Quasi-Polynomial Delay

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Special Section: Representing and Managing Uncertainty
Track: Abstracts

Abstract. The solution set of a fuzzy relation equation (FRE) can be characterized, under mild hypotheses, in terms of its greatest solution and its minimal solutions. The verification of the solvability of a FRE and the computation of its greatest solution are straightforward tasks. Nevertheless, in general, the number of minimal solutions of a FRE increases exponentially with the size of the relations, which makes their computation a hard problem.

This work presents the main results published in the paper "Lobo, D., Medina, J., Merkl, T. C., Pichler, R. (2025). Minimal solutions of fuzzy relation equations via maximal independent elements. Information Sciences, 690, 121558." Namely, there exists a one-to-one relation between minimal coverings of FRE, which characterize minimal solutions, and maximal independent elements of (hyper-)boxes. As a consequence, efficient enumeration methods for maximal independent elements of (hyper-)boxes can be directly applied for the enumeration of the minimal solutions of a FRE. In particular, it is shown that the enumeration of the minimal solutions of a FRE can be done with incremental quasi-polynomial delay.

Keywords: Fuzzy relation equations; Minimal solutions; Enumeration; Independent sets;

Federated Learning with Strategy for Uncertainty Management in Machine Learning Based on Interval-Valued Fuzzy Sets and Choquet Integral for Medical Diagnostic

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Special Section: Representing and Managing Uncertainty
Track: Abstracts

Abstract. This study introduces a federated learning framework specifically designed to address diagnostic challenges arising in privacy-sensitive and uncertainty-rich environments. Such challenges are notably prevalent in medical applications, including breast cancer diagnosis, where datasets are typically limited, imbalanced, or affected by noise, and stringent data privacy regulations prohibit conventional data-sharing methods. Federated learning provides a practical alternative by enabling multiple institutions to collaboratively train machine learning models without directly sharing local datasets. Our methodology emphasizes horizontal federated learning, where individual participants independently train local models and periodically exchange parameters with a central aggregator, thereby iteratively improving global model performance. To systematically address inherent uncertainties, our approach integrates interval-valued fuzzy set theory and the Choquet integral. Specifically, we propose a novel technique for generating interval-valued weights embedded within logistic regression models, explicitly representing the variability and imprecision intrinsic to clinical data. The Choquet integral further enhances this strategy by effectively modeling interdependencies among input features and incorporating uncertainty measures within the aggregation process. Presented experimental results demonstrate that explicitly embedding uncertainty into model parameters significantly improves robustness, particularly in contexts characterized by limited, imprecise, or sensitive data. Consequently, the proposed federated learning framework emerges as particularly effective for healthcare applications, where balancing data privacy requirements with rigorous uncertainty management remains critically important.

Keywords: Federated Learning; Uncertainty Modeling; Machine Learning; Interval-valued Fuzzy Sets; Entropy; Choquet Integral.

OFWA Operators for Multi-Criteria Decision Making

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Special Section: Representing and Managing Uncertainty
Track: Abstracts

Abstract. In the field of multi-criteria decision making, the need to aggregate diverse expert opinions and data with uncertainty has led to the development of increasingly flexible aggregation tools. One such tool is the well-known class of Ordered Weighted Averaging (OWA) operators, which use a list of weights to determine how each value, sorted from greatest to least, influences the overall decision. However, OWA operators lack the flexibility to change these weights depending on the input values. To address this issue, the Ordered Functional Weighted Averaging (OFWA) operators were introduced in "Yager, R.R., Medina, J., (2021) OWA operators with functional weights. Fuzzy Sets and Systems, 414, 38-56.". This class of operators extends the traditional OWA operators by allowing the weights to dynamically adapt to the values being aggregated, thus enabling more precise and context-sensitive aggregation.

This work presents the principal results recently published in the paper "Aragon, R.G., Medina, J., Molina-Ruiz, S., Yager, R.R., (2025) On the complete lattice structure of ordered functional weighted averaging operators. Mathematics, 13(5), 795.", where the algebraic structure of the whole set of OFWA operators is deeply studied. Specifically, it is proven that the collection of all OFWA operators forms a complete lattice with the pointwise ordering, an algebraic property that does not hold for the class of OWA operators. This structure

allows for the computation of both the supremum and infimum of arbitrary families of OFWA operators, an essential feature for scenarios involving multiple expert opinions. In particular, this lattice framework is used to support the creation of new OFWA operators that capture either a conservative perspective (via the infimum) or an aggressive one (via the supremum), thus enabling meta-aggregation of diverse decision strategies.

Keywords: Aggregation operators; Multi-criteria decision-making; Ordered weighted averaging operator.

Embedding-Based Similarity Measure for IVFS

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Special Section: Representing and Managing Uncertainty
Track: Abstracts

Abstract. Fuzzy sets (FSs), widely used to model imprecision, assign a precise membership degree to every element. This can be a limiting constraint in certain contexts. To address this limitation, interval-valued fuzzy sets (IVFSs) were introduced, allowing decision-makers to express membership degrees as a range of possible values rather than a single exact number. Thus, IVFSs offer a more flexible and effective approach to modeling human behavior compared to traditional FSs. Measuring the similarity between two IVFSs is crucial for better understanding and comparing the uncertainty they represent. Similarity measures are fundamental tools for quantifying how alike two objects are, and the literature contains numerous references to these functions, including similarity measures for IVFSs based on the partial order defined by inclusion [1]. However, the existing definition of similarity for IVFSs in the literature is not well-suited to the context addressed in this work because the inclusion relation defined for these sets does not align with the inherent inclusion structure associated with embeddings. In this work, we present an alternative definition of similarity measure for IVFSs based on a different partial order, specifically embedding [2], and propose two methods for its construction. Additionally, we examine the relationship between these methods, identifying cases where they yield the same similarity measure.

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Keywords: Similarity; Interval-Valued Fuzzy Set; Embedding; Aggregation Function.

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GranFRRI: Improving Fuzzy Rough Rule Induction with Granular Approximations

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Special Section: Representing and Managing Uncertainty
Track: Abstracts

Abstract. Rule induction algorithms are valuable tools for generating human-interpretable models from data, particularly in domains where transparency is crucial. Our previous work introduced FRRI (Fuzzy-Rough Rule Induction), a novel algorithm leveraging fuzzy set and rough set theories to induce concise and accurate rule-sets. FRRI utilizes the fuzzy rough lower approximation to assess rule confidence. However, data inconsistency, common in real-world datasets, can challenge the robustness of this approach. Granular approximations offer a more nuanced method for handling inconsistency by minimizing the cost of resolving conflicts based on a chosen loss function. This paper introduces GranFRRI, an extension of FRRI that replaces the fuzzy lower approximation with granular approximations. This integration facilitates novel mechanisms for handling inconsistency, including data relabeling based on granular membership and certainty-based filtering of rule candidates. Through extensive experiments on benchmark datasets, we demonstrate that GranFRRI, particularly when configured with the Mean Squared Error loss function, restricted relabeling, and appropriate certainty thresholds, significantly improves classification accuracy with shorter rules when compared to the original FRRI, while maintaining comparable ruleset sizes.

Keywords: Fuzzy rough set theory; Rule induction; Classification; Granular approximations.

New Approach to the Uncertainty Degree for φ-Intuitionistic Values

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Special Section: Representing and Managing Uncertainty
Track: Abstracts

Abstract. In this contribution, we extend the framework of φ-intuitionistic values by proposing a generalised approach to measuring the degree of uncertainty. In the classical intuitionistic fuzzy setting introduced by Atanassov [1], the uncertainty degree corresponding to a value (u,v) is given by the linear expression $1-u-v$. This formulation is naturally adapted in extensions such as the Pythagorean[2] and q-rang[3] values, where the uncertainty is expressed as $\sqrt{1-u^2-v^2}$ and $\sqrt[q]{1-u^q-v^q}$, respectively. The proposed approach for universal φ-intuitionistic fuzzy sets is a more general one, encompassing these specific cases as instances of a broader family of functions. A phi-intuitionistic fuzzy value[4] is a pair (u, v), where u denotes the degree of membership and v the degree of non-membership, satisfying the condition $\varphi(u)+\varphi(v)\leq 1$ with φ being an automorphism of the unit interval, i.e., a strictly increasing bijection. This definition recovers the classical intuitionistic values when $\varphi(x)=x$, the Pythagorean values when $\varphi(x)=x^2$, and the q-rang values when $\varphi(x)=x^q$. Therefore, the φ-intuitionistic values serve as a unifying generalisation that shows the isomorphism between these previously studied cases. We propose an uncertainty degree as a function π assigning a real number in $[0,1]$ to each φ-intuitionistic value (u, v). This function is required to satisfy the following axioms:

1. $\pi(0,0)=1$,
2. π is decreasing in each variable, and
3. $\pi(u,v)=0$ whenever $(u,v)=(1,0)$ or $(0,1)$.

These axioms ensure that the uncertainty is maximal when both membership and non-membership are absent, and vanishes when the information is crisp. This approach enriches the original idea of uncertainty degree, which served as a sort of residual value whenever the degree of membership and non-membership did not sum up to one, by allowing the user to consider values such as (0.5,0.5) as not certain. Naturally, the construction of such function consists of two parts: choosing a binary aggregation function A, i.e., an increasing function that is fixed to 0 at the value (0,0) and to 1 at values (1,0) and (0,1) and transforming it with a negation function n, i.e., strictly decreasing function that holds $n(0) = 1$ and $n(1) = 0$, resulting in the formula $\pi(x, y) = n(A(\varphi(u), \varphi(v)))$. Then, the degree of uncertainty of classical intuitionistic fuzzy sets can be reconstructed by choosing $A(u, v) = u + v$ and $n(x) = 1 - x$. As an example that does not satisfy the condition $\varphi(u) + \varphi(v) = 1$ whenever $u + v = 1$, one can consider combining the probabilistic sum $A(u, v) = u + v - u * v$ with the standard negation $n(x) = 1 - x$. This general form encompasses the classical, Pythagorean, and q-rang cases, while also allowing for different forms of uncertainty. Some examples of such functions are discussed.

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Keywords: Intuitionistic fuzzy sets; Degree of uncertainty; Generalisation.

Credibility of Decision Rules as a Tool for Decision-Making in Fuzzy Rough Set Theory

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Special Section: Representing and Managing Uncertainty
Track: Abstracts

Abstract. Classification of new objects is an important research area in Fuzzy Rough Set Theory which plays a key role in decision-making [1]. This classification is carried out by using decision rules, which are useful to describe relational datasets and extract information from the new objects to be classified. A fundamental step in the classification process is the choice of consistent decision rules in order to provide robust decisions. The credibility is a relevance indicator of decision rules [3], which computes the consistency of the rules, taking into account the information stored in the given dataset. This work provides two classification methods of new objects based on an extension of the notion of credibility to the multi-adjoint framework [2]. Therefore, the decisions made by these methods are supported by consistent decision rules and, consequently, by reliable information.

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3 J. Stefanowski and A. Tsoukiás. Induction of decision rules and classification in the valued tolerance approach. In J. J. Alpigini, J. F. Peters, A. Skowron, and N. Zhong, editors, Rough Sets and Current Trends in Computing, pages 271–278, Berlin, Heidelberg, 2002. Springer Berlin Heidelberg.

Keywords: Fuzzy Rough Set Theory; Decision rules; Classification methods; Credibility.

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Part XIV
Soft Computing, Uncertainty and Imprecision in Image Processing

Improving Early Warnings of Flash Floods Using Radar Images and AI

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Special Section: Soft Computing, Uncertainty and Imprecision in Image Processing
Track: Abstracts

Abstract. The precipitation is a weather phenomenon that impacts our daily lives in relation agriculture, water management and social activities. However, an excessive amount of rain in a short period of time can cause floods, this is especially dangerous in summer months, where the probability of convective rains is higher.

A widely known episode of rain flash flood is the one of the city of Valencia in Spain that happened the 28 of October 2024, which left more than 200 deaths and a huge amount of destruction. With our work we want to improve the predictability of these episodes by providing a different approach with the use of the reflectivity weather radar images and artificial intelligence.

Reflectivity weather radar is a ground sensor that estimates the amount of rain that is falling from the clouds by sending radio waves around it that are reflected by the drops of water. The Spanish network of radars generates images that cover a circular area with a diameter of 480 Km and has a spatial resolution of 1Km^2 per pixel and a temporal resolution of 10 minutes, which is higher than what most satellite products can offer. We have developed a neural network based on diffusion models that is able to make predictions of the evolution of radar images in the following 90 minutes using the previous 90 minutes of images and the direction and intensity of the wind as input data. This is called nowcasting in the literature.

This model generates overall good predictions, however, in order to build a more robust product that could be used in practical scenarios, we need to account for the uncertainty in the predictions. This can be done by generating different possible outcomes for the same scenario and establish probabilities of occurrence. To do that, our approach is to train different nowcasting models specialized in specific types of storms. We have studied five types of storms: fast-storms (low risk), slow or stationary storms (medium risk), convective trains (high risk), super-cells (high risk) and squall-lines (high risk). The first two types are the most frequent and the last 3 have low-rate occurrence, so the dataset is unbalanced.

A meteorologist expert has done a first manual labelling of 5 years of radar images. We want to extrapolate this labelled dataset to the remaining unlabelled data. This is a semi-supervised classification problem with sequences of images and unbalanced classes. The labelling process is being done with the use of two different approaches. The first one being the use of convolutional neural networks (CNN) to extract spatial-temporal features of the images and generate a membership value for every possible label. The second one is a new proposed method, which is based on the K-nearest-neighbours (KNN) algorithm that labels images using Restricted dissimilarity functions as a distance metric.

The CNN method is more useful at assist the manual labelling process when the number of labelled data is low, but it has difficulties with examples that have prediction values around the decision threshold, as there is a high level of uncertainty. Our proposed method provides a more defined decision method thanks to the vote procedure, increasing the sharpness of the prediction. Additionally, due to the nature of being a different approach, it is able to correctly classify examples that are confusing to CNNs, so both methods can complement each other to improve accuracy. However, it needs a high amount of labelled data to give a reasonable level of accuracy. We are gradually increasing the amount of labelled data, so we expect this method to be improving over time.

Keywords: Uncertainty in meteorology; Semi-supervised classification; Flood risk management.

Evolved Image Filtering Using Aggregation Operators

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Special Section: Soft Computing, Uncertainty and Imprecision in Image Processing
Track: Abstracts

Abstract. Image filtering is the most usual procedure to represent local operations on images. Fundamentally based on function convolution, it has remained a key tool for image processing, and has been a cornerstone for very diverse tasks, from image regularization to feature extraction.

Due to its original inspiration, image filtering is rooted on integration/summation (depending on whether the image positions are considered real- or integer-valued) and multiplication. Hence, the filter (also, kernel) expresses the weights of neighbouring positions around a certain pixel. This has been sufficient for many procedures to be presented over decades, from differentiation to smoothing, eventually leading to the creation of scale-spaces. However, it is relatively restrictive, and some authors have proposed alternative, more flexible interpretations of filterings. A relevant case would be bilateral filtering, which was originally conceived for content-aware smoothing, but can be applied to many other tasks. While not studied as part of any framework, literature also contain filtering schemata that do not abide by classical filtering, e.g. median or max filtering, very popular in image denoising and convolutional neural networks, respectively.

In this work, we study a generalization of image filtering sustained on the parallellisms between classical models for image filtering and weighted means. From that initial standpoint, we elaborate on the accomodation of different aggregation operators to our new modelling filter.

Keywords: Image processing; Aggregation Operators; Convolution; Filtering.

Laplacian-Guided Keypoint Selection for Efficient Fuzzy Transform Approximation in Bayesian Networks

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Special Section: Soft Computing, Uncertainty and Imprecision in Image Processing
Track: Abstracts

Abstract. The appropriate representation of high-dimensional data is the main focus of machine learning, pattern recognition and computer vision. With the same motivation, the F-transform uses fuzzy partitions in order to establish a reduced (compressed) representation of data. Two distinguished properties of the F-transform: the best approximation in a local sense and dimensionality reduction contribute to the fact that the F-transform successfully copes with high-dimensional data representation [1]. In this contribution the efficiency (computational complexity) of the F-transform is the main focus. In our previous research we showed that efficiency is directly connected with the choice of a fuzzy partition, i.e. with the choice of a number and node positions of what which we call basic functions. We propose to apply the approach that identifies keypoints in the data and identify them with the nodes of a fuzzy partition. The technique based on the Laplace-Beltrami operator (Laplacian in short) is suitable for this purpose [2].

In the terminology of F-transform, the Laplacian operator essentially measures how a function differs from its inverse F-transform. The computational advantages of this approach are substantial. By reducing data representation using key Laplacian-identified nodes, we decrease computational complexity from $O(n)$ to $O(k)$, where k is the number of keypoints and typically $k \ll n$. This methodology has proven valuable across multiple domains, including image processing for feature extraction [2], spatial statistics for anomaly detection, and sensor networks for optimal placement of monitoring devices. In each case, the Laplacian-guided keypoint identification preserves critical information while significantly reducing data volume.

Furthermore, theoretical error bounds can be established for this approximation approach. These bounds provide guarantees on the minimum information loss when representing the original data using keypoint-based F-transform, allowing practitioners to make informed decisions about the trade-off between computational

efficiency and approximation accuracy. The approach also enables adaptive node placement strategies, where computational resources are concentrated in regions where the function changes rapidly (indicated by high Laplacian values) while using fewer nodes in smoother regions. This adaptive allocation ensures that information-rich areas receive appropriate representation without wasting resources on regions with minimal variation.

Applying this knowledge could help make training of Bayesian networks more efficient by reducing data complexity while preserving informational value.

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Keywords: Fuzzy transform; Laplacian; Approximation; Bayesian networks.

Part XV
Soft Methods in Statistical Inference and Data Analysis

Resampling Approaches for Multivariate Random Interval Numbers

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Special Section: Soft Methods in Statistical Inference and Data Analysis
Track: Papers

Abstract. The classical bootstrap and similar methods are widely used to solve statistical problems. In this paper, we consider four resampling methods specially tailored for the data consisting of many interval-valued variables. Two of these approaches are generalizations of the classical and smoothed bootstrap for such a particular case. The following two are non-parametric approaches that take into account possible dependencies within intervals for each variable and between the variables themselves. Using numerical simulations, various error measures, and statistical tests, these algorithms are compared to check their overall quality. It seems that particularly one of the considered resampling methods gives valuable bootstrapped samples.

Keywords: Interval-valued numbers; Bootstrap; Imprecise random values; Numerical simulations.

A Soft Clustering Method Derived from the Probabilistic Interpretation of Fuzzy C-Means

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Special Section: Soft Methods in Statistical Inference and Data Analysis
Track: Papers

Abstract. The Bayesian interpretation of Fuzzy C-Means (FCM) opens the door to novel extensions of this technique for soft data clustering. In this paper, we propose a new method based on the minimization of negative log-likelihood of data samples, using basin-hopping and Powell method for minimization. Differently from FCM, we observed a more robust arrangement of prototypes, which could enable the assessment of the suitability of the chosen number of clusters. Moreover, the method enables the data-driven derivation of the fuzzification coefficient, which is directly related to the variance of the component densities. Experiments on synthetic data aim at comparing the proposed method with FCM, by showing benefits and limitations of the two techniques.

Keywords: Fuzzy C-Means; Soft clustering; Maximum likelihood estimation; Fuzzification coefficient.

FLIRT—An Algorithm to Enhance a Regression Model with Federated Learning and GAN-Based Resampling

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Special Section: Soft Methods in Statistical Inference and Data Analysis
Track: Papers

Abstract. Federated learning is a useful method to combine efficient data analysis with privacy protection of data, especially when legal regulations, privacy concerns, or competitive threats restrict data sharing between entities (clients). Instead, clients collaborate to solve the main problem through targeted updates coordinated by some central server. This paper proposes a novel approach combining federated learning, imputation phase, and resampling step to solve the regression problem. The MissForest method is employed during the imputation phase, while the resampling step utilizes an ML algorithm (GAN). The proposed approach is evaluated numerically on various synthetic and practice-oriented datasets. Subsequently, the associated errors related to the statistical properties of the regression models are measured and compared. The results demonstrate that the final model obtained through the proposed method outperforms client-based models in terms of these error metrics.

Keywords: Incomplete data; Model merging; Simulations; Machine learning; Imputation; GAN.

Modeling Treatment Effect with Fuzzy Data

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Special Section: Soft Methods in Statistical Inference and Data Analysis
Track: Papers

Abstract. The treatment effect is a universal concept used in many fields. Regardless of the context, it is crucial to define the measure of the success of the intervention and the method of assessing the effect, which may require the use of various statistical methods. An additional challenge for analysts may be caused by imprecise data on the basis of which they are to assess the size of the treatment effect. It turns out that although the use of fuzzy sets to model imprecise data is something natural, further and deeper analysis of the treatment effect based on such data entails many difficulties. This contribution discusses how to deal with some these problems.

Keywords: Treatment effect; Fuzzy numbers; Fuzzy random variables; Generalized Hukuhara difference; Estimation.

Alpha-Maxmin Classification with an Ensemble of Structural Restricted Boltzmann Machines

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Special Section: Soft Methods in Statistical Inference and Data Analysis
Track: Papers

Abstract. This article addresses a classification problem relying on an ensemble of Structural Restricted Boltzmann Machines (SRBMs). Each SRBM in the ensemble is trained by imposing structural constraints on the related weight matrix, so as to enforce sparsity, and results in a probabilistic classifier. Hence, given a new instance, the ensemble gives rise to a credal classifier where the classification is carried out relying on the alpha-maxmin criterion, depending on a pessimism index $\alpha \in [0, 1]$, and a β-quantile filtering of outliers. The paper presents an experimental analysis on artificial data sets to highlight the role of the parameters α and β in the classification performances.

Keywords: Structural Restricted Boltzmann Machine; Ensemble; Classification; Alpha-Maxmin.

Fuzzy Events Require a Strengthened Form of Independence

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Special Section: Soft Methods in Statistical Inference and Data Analysis
Track: Abstracts

Abstract. Most of classical statistical results (like the central limit theorem, laws of large numbers, etc.) are formulated for sequences of independent events. Usually, independence of events A, B means that the conditional probability is the same as the unconditional one, $P(A|B) = P(A)$. The conditional probability is defined using the probability of the intersection of A and B. For fuzzy events, we have many possible fuzzy intersections which may lead to different formulations of independence. Several choices were treated in previous works and generalizations of classical theorems were successfully derived.

We put additional requirements on independence of events. If A, B are independent, we expect also independence of the complements. However, this natural condition is not always satisfied in the approaches found in the literature. As we can hardly imagine statistics build without it, we say that events A, B are strongly independent if $P(A|B) = P(A)$, $P(A'|B) = P(A')$, $P(B|A) = P(B)$, and $P(B'|A) = P(B')$.

The next question is which fuzzy event structures possess non-trivial examples of strongly independent observables (generalized random variables). The main result is that this is no problem when we use the product intersection. For other types of fuzzy intersections, strong independence appears to be a very strong and restrictive assumption. Thus it allows to prove analogs of the theorems, but they apply only to very special cases.

In the case of MV-algebras, our conclusion is the following: The classical theory extends naturally to MV-algebras with product [1]. However, in MV-algebras without product, we do not see any satisfactory form of independence. Thus the presence of a product (and its use in the definition of conditional probability and independence) appears crucial for a successful development of probability and statistics using fuzzy events.

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Keywords: Probability; Statistics; Tribe; MV-algebra; Conditional probability; Independence.

A Proposal for Measuring Polarization in the Context of Ordinal Scales

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Special Section: Soft Methods in Statistical Inference and Data Analysis
Track: Abstracts

Abstract. How to measure polarization in a society is a problem that has received attention from researchers from different disciplines and approaches. In this contribution, we focus on the case of individuals of a society evaluating an issue through an ordinal scale formed by linguistic terms (e.g., 'very bad', 'bad', 'fair', 'good', 'very good' and 'excellent'). Sometimes these scales are not uniform, in the sense that the psychological distances (or ordinal proximities) between consecutive terms are not perceived as identical (for instance, if 'very good' is perceived as closer to 'excellent' than to 'good'). To manage non-uniform ordered qualitative scales in a purely ordinal way, García-Lapresta and Pérez-Román (Applied Soft Computing, 2015) introduce the notion of ordinal proximity measure.

In this context, we introduce polarization measures as functions that assign a number between 0 and 1 to each profile of individual assessments, satisfying some conditions. Among this family of polarization measures, we propose a parameterized class of functions that aggregate the ordinal proximities between the assessments of each pair of individuals and the highest and lowest terms of the ordinal scale, following the approach of Guevara, Gómez, Robles and Montero (IPMU, 2020) in a fuzzy framework.

This aggregation process is managed through different power means in several steps. We analyze the role of the parameters that provide suitable polarization measures.

Keywords: Polarization; Ordinal scales; Ordinal proximity measures; Power means.

A Hierarchical Representation of Fuzziness in Epistemic Fuzzy Data Analysis

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Abstract. Statistical data analysis often involves managing multiple levels of uncertainty, particularly in research fields where random and non-random factors influence observations. This is evident, for instance, in social and economic surveys, where variability arises not only from sampling fluctuations but also from subjective responses or measurement imprecision [6,7]. In such cases, fuzzy set theory provides a powerful framework to account for these different forms of uncertainty by modeling epistemic imprecision alongside traditional stochastic variability [3,5]. This study builds on the concept that fuzzy data can be interpreted as the outcome of a conditional probabilistic model. We propose a comprehensive approach that explicitly integrates the epistemic mechanism governing the generation of fuzzy data [2]. By adopting a structure of hierarchical generalized linear model, our approach establishes a direct link between the observed characteristics of fuzzy numbers (such as mode or spread) and the latent statistical model that describes the underlying, non-fuzzy sample before fuzzification [1]. This formulation allows for a broader data representation, in which the unobserved true value belongs to a plausible range rather than a precise point. While formally distinct, this perspective closely aligns the interpretation of epistemic fuzziness with that offered by statistical coarsening. In this contribution, we

examine two hierarchical representations of fuzziness. In the first, the true, non-fuzzy random variables are assumed to be independent of the fuzzification mechanism. In the second, the fuzzification process interacts with the underlying random model, leading to a more complex and data-dependent uncertainty structure, where the stochastic properties of the latent variables influence the degree of fuzziness. While the first approach offers flexibility at the cost of simplicity, the second introduces a higher-order hierarchical relationship among the model components. We investigate the properties of both epistemic representations and demonstrate their effectiveness in capturing empirical phenomena through real-world case studies.

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Keywords: Fuzzy statistics; Hierarchical Bayesian models; Regression-like models.

On the Consistency of Fuzzy Linguistic Summaries with Multiple Qualifiers

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Special Section: Soft Methods in Statistical Inference and Data Analysis
Track: Abstracts

Abstract. Fuzzy linguistic summaries (FLS) are sentences that allow for describing patterns about large numerical datasets in natural language. To construct them, we use linguistic expressions, for instance, "Most employees with small experience are young and earn a low salary"). Fuzzy linguistic summaries consist of: - a linguistic quantifier (e.g., most, about half), - a summarizer and a qualifier - both are some attributes with linguistic values (e.g., low salary, small experience). In this contribution, we consider the recently introduced general definition of FLSs and focus on cases where the number of summarizers and qualifiers is at most two. Additionally, we consider various aggregation functions for connectives and investigate the property of consistency. We say that an FLS is consistent if it satisfies the properties of non-contradiction and double negation. Both are related to the truth function and fuzzy negations, which must be used to calculate the values of this function for contradictory forms of FLSs. We present properties of linguistic quantifiers, conditions for fuzzy negations, and other necessary fuzzy connectives to ensure consistency.

Keywords: Fuzzy linguistic summary; Fuzzy negation; Consistency; Double negation;

Implicational Quantifiers for Fuzzy Association Rule Mining: Theory and Application to Bipolar Disorder Data

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Abstract. Implicational quantifiers constitute a specific subclass of general unary hypothesis automaton quantifiers [1]. In this work, we investigate their key properties, focusing primarily on monotonicity and representability. We analyse how these properties influence the quantifier's behaviour in the context of fuzzy set operations. Building upon this theoretical foundation, we apply the developed quantifiers to generate association rules from datasets collected from smartphones of bipolar disorder patients. The proposed approach is promising for extracting meaningful patterns in a complex, uncertainty-laden environment, thus supporting psychiatrists engaged in remote mental health monitoring.

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Keywords: Implicational quantifiers; Generalized quantifiers; Association rule mining; Mental health monitoring.

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Part XVI
The Role and Value of Information in Decision Making

Pollution Control under Uncertainty: Integrating Optimal Control Theory and Value of Information

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Special Section: The Role and Value of Information in Decision Making
Track: Abstracts

Abstract. Environmental management decisions are often made under substantial uncertainty regarding system responses to various stressors, including anthropogenic activities. This uncertainty can hinder effective decision-making, leading to suboptimal or delayed actions. Value of Information (VoI) theory provides a decision-analytic framework to quantify the benefits of reducing uncertainty through information acquisition. While VoI has been widely applied in static settings, real-world environmental challenges are inherently dynamic, requiring sequential decision-making over time.

In this study, we develop a dynamic framework for evaluating VoI using an optimal control (OC) theory approach. We apply this methodology to a pollution emission control model inspired by previous literature. Unlike traditional static VoI analyses, our approach incorporates uncertainty in key system parameters and initial conditions while allowing for continuous-time decision-making. Specifically, we examine two cases of uncertainty: (i) uncertainty in the initial pollution stock and (ii) uncertainty in the environmental absorption rate. We derive optimal control strategies under different uncertainty scenarios and compare them to a naive approach where the decision-maker assumes average parameter values. We introduce the concept of the "payoff gap," which quantifies the benefit of resolving uncertainty over time.

Our results demonstrate that incorporating VoI within a dynamic OC framework leads to more informed and effective environmental policies. By quantifying the benefits of resolving uncertainty, our approach provides actionable insights for policymakers in managing pollution control and other dynamic environmental challenges. To our knowledge, this is one of the first studies to integrate OC theory with VoI analysis, offering a novel methodological contribution that can be adapted to other dynamic environmental systems.

Keywords: Environmental management; Value of Information; Uncertainty; Optimal Control; Pollution Control; Decision-making; Dynamic Systems.

Prioritized Preference Aggregation for Non-Uniform Groups of Agents

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Special Section: The Role and Value of Information in Decision Making
Track: Abstracts

Abstract. A novel approach to the important problem of aggregation of individual fuzzy preferences of a group of agents in a group decision process is presented. It is different than the traditional, widely employed aggregation based on the averaging, the weighed conjunction/disjunction, etc. of the respective preference degrees between pairs of options which stand for the uniformity of the group of agents in the sense that all testimonies of the agents are to be accounted for, even if the agents (and maybe also options) are assigned various importance degrees. The novel approach proposed here assumes that the group is not uniform in the above sense which implies that the testimonies of the most important agents are crucial and decisive, so that they should be

followed, possibly ignoring testimonies of lower level agents, as it often happens in business or the military. Yager's approach to the so-called prioritized aggregation is employed. The results are very promising and can yield new vistas and perspectives for (fuzzy) preference based group decision making in complex groups of agents, notably aythoritarian hierarchies.

Keywords: Group decision making; Fuzzy preference Fuzzy preference aggregation; Prioritized aggregation.

Extending the Concept of the Value of Information to Fuzzy Decision-Making with Expert Systems

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Special Section: The Role and Value of Information in Decision Making
Track: Abstracts

Abstract. The Value of Information (VoI) is a concept in decision theory that quantifies the benefit of obtaining additional information before making a decision under uncertainty. It indicates how much improvement in decision-making can be expected when extra knowledge becomes available. This concept is particularly relevant when the outcome of a decision is described by a profit (or utility) function. When the variables in the profit function are random, the VoI is calculated as the difference between two values: the expected optimal profit assuming the realization of the random variables are known, and the optimal expected profit when decisions are made based only on the probability distribution of the random variables. We propose an alternative approach for situations where uncertainty is represented by fuzzy sets rather than by probability distributions. This setting is particularly suitable when precise statistical information is unavailable or unreliable, and the decision-making process relies on qualitative assessments provided by experts. In our examples, solving the value of information problem, we are dealing both with imprecise information and with systems when experts express their opinions linguistically. In the second case, expert knowledge is encoded as a set of fuzzy rules that describe relationships between inputs and outputs in a form that mirrors natural language.

Keywords: Value of Information; Decision making; Expert system.

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